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The out-of-sample forecasting performance of non-linear models of real exchange rate behaviour: The case of the South African Rand*

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Abstract

This paper analyses the out-of-sample forecasting performance of non-linear vs. linear models for the South African rand against the United States dollar and the British pound, in real terms. We compare the forecasting performance of point, interval and density forecasts for non-linear Band-TAR and ESTAR models to linear autoregressive models. Our data spans from 1970:01 to 2012:07, and we found that there are no significant gains from using either the Band-TAR or ESTAR non-linear models, compared to the linear AR model in terms of out-of-sample forecasting performance, especially at short horizons. We draw similar conclusions to other literature, and find that for the South African rand against the United States dollar and British pound, non-linearities are too weak for Band-TAR and ESTAR models to estimate.

JEL classifications: C22, C52, C53, F31, F47.

Keywords: Real exchange rate; Transaction costs; Band-threshold autoregressive model; Exponential smooth transition autoregressive model; Point forecast; Interval forecast; Density forecast; South Africa.

1. Introduction

Two of South Africa's main trading partners are the United States and the United Kingdom. The size of these two economies alone result in greater volatility of the South African exchange rate in terms of these two currencies. Large fluctuations in real exchange rates have potential trade balance and policy implications. According to Schnatz (2006), it is not necessarily the level of the real exchange rate, but rather the movement towards or away from some long-run equilibrium that makes planning and policy making a challenge. It therefore becomes imperative to be able to accurately

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forecast real exchange rates, in an attempt to remove some of the uncertainties in decision- and policy making.

Internationally there has been a drive towards estimating real exchange rate behaviour using non-linear models. These are well motivated by theoretical models, developed by Obstfeld and Rogoff (2000), incorporating transaction costs (transportation costs, tariffs and nontariff barriers, as well as any other costs that agents incur in international trade). Intuitively, transaction costs give rise to a band of inactivity where arbitrage is not profitable, so that real exchange rate fluctuations are not corrected inside of the band. However, arbitrage works to bring the real exchange rate back to the edge of the band if the real exchange rate moves outside of the band.

In line with the theoretical models involving transaction costs, one can characterize real exchange rate movements based on a Band-Threshold Autoregressive (Band-TAR) and exponential smooth-transition autoregressive (ESTAR) models. The Band-TAR model is characterized by unit-root behavior in an inner regime and reversion to the edge of the unit-root band in an outer regime. In contrast to the discrete regime switching that characterizes the Band-TAR model, the ESTAR model allows for smooth transition between regimes. As pointed out by Rapach and Wohar (2006), non-synchronous adjustment by heterogeneous agents and time aggregation are likely to lead to smooth switching of regimes, rather than discrete switches, and this is more likely to be the case for real exchange rates, since they are based on broad price indices.

Against this backdrop, this paper follows the methodology of Rapach and Wohar (2006) and implements Band-TAR and ESTAR models to estimate the non-linear behaviour of real exchange rates within sample as well as out-of-sample for the South African real exchange rate against the US dollar and British pound. The non-linear out-of-sample point, interval and density forecasts are evaluated relative to the corresponding out-of-sample point, interval and density forecasts from the linear AR model. We use the modified M-DM statistic of Harvey et al., and the weighted version of M-DM statistic (MW-DM) developed by van Dijk and Franses (2003) to determine whether the non-linear AR models' point forecasts are superior to the linear AR models' forecasts. Wallis (2003) expanded on the likelihood ratio tests for independent (χ^2_{IND}), conditional (χ^2_{CC}) and unconditional (χ^2_{UC}) coverage developed by Christoffersen (1998). These tests are used to compare interval forecasts of non-linear AR models to those of linear AR models. To compare density forecasts, we apply the Doornik and Hansen (1994), DH statistic, Kolmogorov-Smirnov (KS) statistic and the Ljung-Box statistic.

Though, there exist a large number of studies that have looked into forecasting nominal exchange rates (primarily with respect to the US dollar) for South Africa,1 the

[1] See for example Brink and Koekemoer (2000), Bonga-Bonga (2008, 2009), Botha and Pretorius (2009), Gupta and Kabundi (2010), Alpanda et al., (2011), de Bruyn et al., (2013forthcoming), and references cited therein, whereby the methods range from the basic monetary model and its extensions, ESTAR

literature on forecasting the real exchange is sparse, to say the least. Most of the studies dealing with real exchange rate in South Africa essentially look at in-sample linear and at times non-linear (Markov-switching) modelling of the real exchange rate based on a wide variety of fundamentals such as interest rate differentials, suitable productivity measures, commodity prices, openness, and fiscal balance and capital flows (see for example, Chinn, 1999; Aron et al., 2000; MacDonald and Ricci, 2004; Mtonga, 2006; Frankel, 2007; Fattouh et al., 2008; Kaufmann et al., (2011); de Jager, 2012; Égert, 2012, and references cited therein). The papers by Fattouh et al., (2008) and Égert (2012) are the only two studies that explicitly look at forecasting the dollar-based South African real exchange rate. While Égert (2012) indicates poor out-of-sample forecasting power of a linear model in terms of prediction of turning points in the actual real exchange rate, estimated using dynamic ordinary least squares (DOLS), based on a wide set of fundamentals, Fattouh et al., (2008) shows that a linear error correction model comprising of fundamentals, produces lower one step-ahead (point) forecast errors than the Markov-switching error correction model.

Given the limited and mostly a preliminary existing literature on forecasting of the real exchange rate in South Africa, we, in this paper, aim to provide comprehensive in-sample and out-of-sample (point, interval and density forecasts-based) evidence of whether one should look to model real exchange rate behaviour in South Africa using nonlinear models. This paper is set out as follows: Section 2 considers both the Band-TAR[2] (as proposed by Obstfeld and Taylor (1997) and implemented by Rapach and Wohar (2006)) as well as the ESTAR (proposed by Taylor et al (2001) and implemented by Rapach and Wohar (2006)). Section 3 presents the point, interval and density forecast evaluations. Section 4 discusses the results. Section 5 provides comparisons between the in-sample conditional densities of the linear AR and the non-linear AR models, while Section 6 concludes.

2. Methodology

We use a similar convention to Obstfeld and Taylor (1997) where the log-level of the real exchange rate, q_t can be expressed as:

and time-varying models, dynamic model averaging, vector autoregressive (VAR) models, Bayesian VAR (BVAR), large-scale VAR models (involving 266 macroeconomic variables) based on factor analysis and Bayesian shrinkage and dynamic stochastic general equilibrium (DSGE) model.

[2] Given that there exists some attempts to model the real exchange rate in South Africa using Markov-switching models, it would have been more natural to use such a model as well in our forecasting exercise. However, given the lack of favorable evidence for such a class of non-linear models in forecasting the real exchange rate in South Africa (Fattouh et al., 2008), and the fact that the Band-TAR model, which also captures discrete jumps in the real exchange rate like the Markov-switching model and is also more theoretically motivated in terms of transaction costs (Rapach and Wohar, 2006), we decided to leave out the Markov-switching AR models from our analysis. The use of MS-AR model in forecasting South African real exchange rates however, can be an interesting area for future research.

$$q_t = s_t + p_t^* - p_t, \quad (1)$$

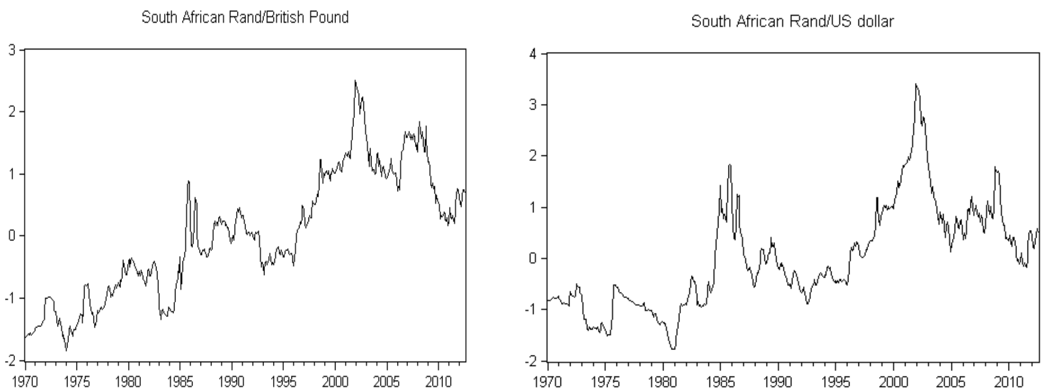
where, s_t is the log-level of the domestic price of the foreign currency and p_t and p_t^* represent the log-level of the domestic and foreign price levels respectively. Furthermore, q_t is demeaned and can be interpreted as a measure of the deviation from PPP.

As mentioned earlier, the presence of transactions costs may imply that q_t follows a random walk, which can then be better modelled by a non-linear process. This implies that the conventional unit root tests of long-run PPP may not be appropriate.

The data for this study was sourced from Global Financial Data, and includes the exchange rates for South Africa's two largest trading partners, the United States and United Kingdom, as well as the consumer price indices for these countries. Our in-sample period is then defined as 1970:01-1994:12 and the out-of-sample period as 1995:01-2012:07. For the latter period we aim to evaluate the forecasting performance of the models based on the point, interval and density forecasts.

South Africa saw large structural changes after the first democratic election in 1994, as the country became more open to trade after sanctions were lifted. Based on this, we use a similar in-sample period (1970:01-1994:12) for South Africa as the post-Bretton Woods period. Furthermore, by using a similar period, comparisons can be drawn between this study and results found by Rapach and Wohar (2006). Data for the Band-TAR model is demeaned or detrended. Data for the ESTAR model is normalised to start from zero in the first period and after allowing for a lag in both models the sample period starts at 1970:02. The descriptive statistics of the data is presented in Appendix 1. The deviation of the two exchange rates from their means is not high. A data series that is normally distributed has a skewness of zero and a kurtosis of 3. The rand-dollar (rand-pound) exchange rate has a skewness of 0.7 (0.2) and kurtosis of 3.4 (2.1) suggesting that none of the exchange rates has normal distribution. This is further confirmed by the rejection of the null of normal distribution of the Jarque-Bera test. The full sample rand-dollar and rand-pound real exchange rates are shown in Figure 1. The figure depicts high volatility and thus suggests that South Africa's exchange rates might have undergone some structural changes during the last 3 decades. The volatile periods coincide with some important economic features and socio-political conditions that prevailed in South Africa. During the early 1970s, a relatively rigid administered system made way for a managed, floating exchange rate regime. Measures aiming at a freely floating commercial rand were introduced following the De Kock Commission Report in 1979. However, the socio-political instability of mid 1980s rendered the rand extremely vulnerable to negative foreign sentiments. The diverse financial and trade sanctions against South Africa resulted into balance of payment crisis. Besides the debt standstill agreement, exchange control on capital transfers by non-residents was re-established in the form of the financial rand. South Africa witnessed the highest depreciation (loss of about 50 per cent in value) of the rand against the US dollar in history due to uncertainty and lack of confidence (Coleman et al., 2011). In 1994, the first democratic election of the Government of Unity was conducted and this restored the country's international financial relations. The financial rand was finally eliminated in March 1995 and barely was there any exchange control over non-residents. From July 1995 onwards, there was a gradual relaxation of exchange control over residents. These measures marked the beginning of complete integration of South Africa into the international capital markets. The financial contagion effect from the Asian crisis in 1997/98 led to a decline in investor confidence in emerging-market economies, including South Africa as the value of rand fall. The rand depreciated between late 2001 and early 2002 but regained its value during 2003 and 2004. It is viewed that this sudden depreciation was due to speculation against the rand and other external factors but not necessarily by economic fundamentals (de Jager, 2012; de Bruyn et al., forthcoming). The recent global financial crisis towards the end of the sample contributed to depressing real exchange rates.

Figure 1. Real exchange rate log-levels, 1970:01-2012:07 for the United States and United Kingdom relative to the South African Rand.



2.1 Band-TAR model

Obstfeld and Taylor (1997) estimated the Band-TAR model to allow for a transaction cost band, where within the band deviations from PPP may display unit root behaviour; while outside of the band the process switches abruptly to a process with no unit root. The Band-TAR model takes on the form:

$$\Delta q_t = \begin{cases} \lambda_{out}(q_{t-1} - c) + \varepsilon_t^{out} & \text{if } q_{t-1} > c; \\ \varepsilon_t^{in} \text{ if} & c \geq q_{t-1} \geq -c; \\ \lambda_{out}(q_{t-1} + c) + \varepsilon_t^{out} & \text{if } -c > q_{t-1}; \end{cases} \quad (2)$$

where q_t is the log-level of the real exchange rate, Δ is the first difference operator, $\varepsilon_t^{out} \sim N(0, \sigma_{out}^2)$ and $\varepsilon_t^{in} \sim N(0, \sigma_{in}^2)$. Real exchange rates follow a random walk inside the band [-c,c] so there is no central tendency; whereas outside the band exchange rates do not convert back to the threshold but to the edge of the band when $\lambda_{out} < 0$. Building on Obstfeld and Taylor (1997) and Rapach and Wohar (2006) we implement maximum likelihood estimation to estimate Equation 2 for the real exchange rates of the United States and United Kingdom with respect to the South African rand. A grid search is used to obtain possible c-values and the outer regime is specified to contain at least 15% of the observations for q_{t-1}.

2.2 ESTAR model

The parsimonious ESTAR can be written as specified by Taylor et al. (2001) and implemented by Rapach and Wohar (2006):

$$q_t = q_{t-1}\{1 - \exp\left[\alpha(q_{t-1} - n)^2\right]\}\,(q_{t-1} - n) + \varepsilon_t \qquad (3)$$

where q_t contains no unit root and is ergodic, $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$, n represents the long-run equilibrium level for q_t. In the ESTAR model, the real exchange rate behaves as a random walk when $q_{t-1} = n$, and the speed of mean reversion increases as the real exchange rate moves away from its long-run equilibrium value (assuming $\alpha < 0$). As α increases in absolute value, the nonlinear effect becomes stronger. . Once again we follow Obstfeld and Taylor (1997) and Rapach and Wohar (2006) and use multivariate non-linear least squares to estimate Equation 3 for the real exchange rates of the United States (U.S dollar) and United Kingdom (Great Britain pound), relative to the South African rand (ZAR).

3. Point, Interval and Density Forecasts

3.1 Forecasts Construction

Our main goal is to identify whether out-of-sample point, interval and density forecasts generated by the Band-TAR and ESTAR models are superior to those forecasted by an AR(1) model. The simple linear AR and the non-linear AR models in Equation 2 and 3 is used to estimate out-of-sample forecasts of q_{t+h} conditional on q_t for $t = R, \ldots, R + P - h$.[3]

[3] R is defined as the first R-amount of in-sample periods for q_t, our out-of-sample period then spans P additional observations for q_t.

To make comparison easier, we first assume that the disturbance terms in the linear and non-linear AR models are Gaussian, and later relax this assumption. Although it is easy to generate point, interval and density forecasts for the linear AR model under this assumption, the same is not the case for non-linear AR models even when $h \geq 2$, as $E[f(x)] \neq f[E(x)]$. For this reason we adopt the simulation-based procedure used in Rapach and Wohar (2006) to generate forecasts for the non-linear AR models.[4] First we consider a more compact version of Equation 3, $q_{t+1} = f(q_t) + \varepsilon_{t+1}$, where $h = 2$. In order to estimate forecasts for a given q_t we simulate realizations of q_{t+1} as $q^*_{t+1} = f(q_t) + \sigma_\varepsilon e^*_{t+1}$, where e^*_{t+1} is randomly drawn from a normal distribution. We then simulate realizations for q_{t+2} as $q^*_{t+2} = f(q^*_{t+1}) + \sigma_\varepsilon e^*_{t+2}$. After repeating the process a thousand times the point forecast is given as the mean of the thousand simulated realizations for a given q_t. When estimating inter-quartile forecasts for q_{t+2}, the 250th and 750th simulated realizations from the sorted 1000 realizations is used. It is straightforward to calculate density forecasts for q_{t+2}, given q_t, using our set of a 1 000 simulated results. This procedure can be applied to generate point, interval and density forecasts for any q_{t+h}, given q_t.

3.2 Point Forecast Evaluation

We evaluate the point forecasts for the Band-TAR and ESTAR models by focusing on the MSFE and use the Diebold and Mariano (1995) procedure to compare models to one another.[5] This procedure tests the null hypothesis that the non-linear AR model has the same predictive ability as the linear AR model, against the alternative that the non-linear AR model has a smaller MSFE. Following Silverstovs and van Dijk (2003) we use the modified M-DM statistic of Harvey, et al. (1997) which adjusts for finite-sample size distortions. We also consider adjusting the weighted statistic using Harvey et al. (1997) to obtain the modified MW-DM statistic. The adjusted MW-DM statistic originated from the weighted Diebold and Mariano (1995) statistic developed by van Dijk and Franses (2003), which attaches different weights to observations in different regimes. We employ the first weight function which attaches a larger weight to observations in both tails of the distribution of q_t. The first weight function is selected given the assumption of symmetric adjustment in ESTAR and Band-TAR models. It can be expressed as $w_T(\omega_t) = 1 - \varphi(q_t)/\max[\varphi(q_t)]$, where$\varphi(q_t)$ is the density

[4] Simulation-based procedures work better for creating non-linear AR forecasts, shown in Clements and Smith (1997).

[5] $MSFE_i = \left(\frac{1}{P_h}\right)\sum_{t=R}^{R+P-h} e^2_{i,t+h|t}$, given $(i = N, L)$and $P_h = P - (h-1)$. The point forecast error at point h corresponds to $e_{N,t+h|t}$ and $e_{L,t+h|t}(t = R, \ldots, R+P-h)$ for the non-linear and linear AR models respectively.

function of q_t.The Student's t distribution with $P_h - 1$ degrees of freedom is used to assess parameter significance.

3.3 Interval Forecast Evaluation

Interval forecasts are evaluated by following Wallis (2003) who based inferences on exact p-values rather than asymptotic distributions of the Pearson χ^2 tests. This allows for more accurate inference.[6]Wallis (2003) expanded on the likelihood ratio tests for independent(χ^2_{IND}), conditional(χ^2_{CC}) and unconditional(χ^2_{UC}) coverage developed by Christoffersen (1998). These statistics are based on variables that indicate whether the actual observation falls inside the interval by taking on a value of one or zero. The indicator variable is one if the actual observation falls inside the interval, and zero if it does not. Good interval forecasts should have good coverage and be independently distributed over time (Christoffersen, 1998).

We then follow the procedure by Siliverstovs and van Dijk (2003), where indicator variables are divided into h independent subgroups, after applying the χ^2_{UC}, χ^2_{CC}, and χ^2_{IND} tests, we can reject the null hypothesis on an overall level if any one of the subgroups can be rejected at the α/h significance level. With this procedure we restrict the maximum amount of subgroups by means of a restriction on the amount of indicator variables in each group. These test statistics are analyzed using contingency tables, comparing observed outcomes to the expected number of outcomes under the appropriate null hypothesis. When $h \geq 2$, we have to modify the procedure for optimal forecasts and account for autocorrelation of order $h - 1$. This is done to prevent indicator variables used in the Pearson χ^2 from exhibiting autocorrelation as well.

3.4 Density Forecast Evaluation

Diebold et al. (1998) evaluate density forecasts using a probability integral transformation (PIT) technique and show that the series is distributed $iid\ U(0,1)$ under the null hypothesis that the density forecast generated is correct. Following Rapach and Wohar (2006), we test for uniformity using the Kolmogorov-Smirnov (KS) statistic.[7] Using Berkowitz (2001), we transform the PIT series to be distributed $iid\ N(0,1)$, this is done by using the inverse of the normal cumulative density function and by assuming that the density forecast is correct. Again following Rapach and Wohar (2006), we test for normality of the PIT series using the Doornik and Hansen (1994), DH statistic.[8] It is important to note that the KS and DH statistics assume independence. To explicitly test

[6] Wallis (2003) describes how to construct asymptotic distributions of χ^2 and exact p-values.
[7] Clements and Smith (2000) and Siliverstovs and van Dijk (2003) also test for uniformity using the KS statistic. Critical values for the KS statistic are calculate using the procedures in Miller (1956).

for independence, we adopt the Diebold et al. (1998) technique and test for autocorrelation in the PITs. Following Siliverstovs and van Dijk (2003), we use the Ljung-Box statistic to test for first -order autocorrelation.

Using the same process as in section 3.3, the indicator variables are divided into h independent subgroups. We then apply KS, DH and Ljung-Box tests to each of the subgroups and once again reject the relevant null hypothesis at overall significance if any one of the subgroups is rejected at the α/h significance level.

4. Empirical Results

4.1 Results from in-sample estimations

Table 1 reports the results from the in-sample parameter estimates for both the Band-TAR and ESTAR models for the rand-dollar and the rand-pound real exchange rates. We also present some diagnostic tests. Important issues of interpretation in the threshold models rest on the speed of convergence and the spread of the commodity points or threshold level as adjustment is characterized by these two parameters. The real exchange rate follows a random walk inside the band, as transaction costs prevent arbitrage from correcting real exchange rate deviations, while outside the band, arbitrage forces the real exchange rate to move back to the edge of the band when $\lambda_{out} < 0$. Panel A shows the results for the Band-TAR model where λ_{out} is negative and statistically significant at 10% for both rand-dollar and rand-pound exchange rates. This finding is similar to those of Obstfeld and Taylor (1997). This implies that the real exchange rate exhibits reversion properties. Thus, when the South Africa real exchange rates deviate from PPP (beyond the bands), then market forces will move the real exchange rates back to PPP. The λ_{out} for the rand-dollar exchange rate is also much smaller in absolute value compared to the rand-pound exchange rate. While the convergence speed is about 8% per period for the rand-dollar exchange rate, it is about 20% per period for the rand-pound exchange rate implying that the later converges about thrice faster than the former. Although casual evidence suggests that the South African rand and the US dollor (UK pound), tend to revert towards each other over time, there are protracted periods in which the exchange rate deviates from its PPP level. How persistent are these deviations? A measure of persistence is the half-life of PPP deviations. From our results, the South African exchange rate deviations have implied half-lives of 8.5 months and 3 months for the rand-dollar exchange rate and rand-pound exchange rates respectively. [9] These half-lives are quite short and indicate that, through the progress of globalization, South Africa has a good ability to enable the

[8] Clements and Smith (2000) and Siliverstovs and van Dijk and Franses (2003) also use this to test for normality.

[9] Half-life is computed as: $\ln(0.5)/\ln(1+\lambda)$.

movements in its exchange rates to accord with purchasing power parity. The estimated threshold is statistically significant for both exchange rates. The threshold value which is essentially a data-based estimate of transaction costs or, more generally, of "band of inaction" is 8.5% for United States and 10.6% for United Kingdom. These values are reasonable given the estimates of transaction costs derived by IMF CIF-FOB ratios of 8.8 to 12. 6 for African countries (Chasomeris, 2009).We use the sample period 1970:01 to 1994:12 for both countries to estimate the Band-TAR model. By including a lag in Equation 1 and 2 we lose one observation. The numbers of observations in the outer and inner regimes are also reported in Panel A of Table 1.

Panel B contains the results of the ESTAR model for the same period as the Band-TAR model. We reject at 10% and 1 % significance level respectively, the restrictions that the equilibrium log-level of rand-dollar and pound-dollar exchange rate is zero $(n = 0)$ concluding therefore that the real exchange rates do not behave as a random walk and is not increasingly mean reverting with the absolute size of the deviation from equilibrium. This is also confirmed by the fact the speed of convergence though negative is not statistically significant neither for the US or UK. The ESTAR model therefore does not depict significant evidence of nonlinear mean reversion of each of the real exchange rates over time. For both the Band-TAR and ESTAR models, we provide a battery of diagnostic tests in Table 1. For both models, we reject the hypothesis of no serial correlation, no ARCH effect and normality. These rejections could be as a result of the fact that we used pre-specified Band-TAR and ESTAR models in line with theory and hence, does not allow for enough flexibility, especially in terms of lag-lengths to capture the true data-generating-process. Based on the AIC and BIC statistics, if there is need to make a choice between Band-TAR and ESTAR models, the former would be the obvious choice.

Table 1: In-sample parameter estimates for the Band-TAR and ESTAR model

South African real exchange rate with respect to the United States dollar and Great Britain pound

	United States	United Kingdom
Panel A: Band-TAR model		
Sample	1970:01-1994:12	1970:01-1994:12
λ_{out}	-0.078 (0.056)	-0.200*** (0.078)
C	0.085*** (0.019)	0.106*** (0.032)
σ_{out}	0.026	0.017
σ_{in}	0.009	0.013
n_{out}	49	59
n_{in}	250	240
log *L*	-4.614	16.363
AIC	-1214.748	-1311.353
BIC	-1211.047	-1307.652
Q(1)	13.302 [0.000]	8.728 [0.003]
Q(4)	17.390 [0.002]	15.176 [0.004]
Q^2(1)	7.715 [0.005]	13.003 [0.000]
Q^2(4)	37.411 [0.000]	22.145 [0.000]
ARCH(1)	7.642 [0.006]	12.881 [0.000]
ARCH(4)	27.944 [0.000]	17.126 [0.002]
Jarque-Bera	1177.284 [0.000]	5.874 [0.015]
Panel B: ESTAR model		
Sample	1970:01-1994:12	1970:01-1994:12
α	-1.468*** (0.581)	-1.973*** (0.731)
η	0.042* (0.021)	0.122*** (0.016)
σ	0.013	0.014
log *L*	405.480	492.360
AIC	-3974.941	-4375.037
BIC	-3966.472	-4366.568
Q(1)	52.695 [0.000]	34.381 [0.000]
Q(4)	56.984 [0.000]	35.868 [0.000]
Q^2(1)	12.902 [0.000]	11.387 [0.001]
Q^2(4)	20.662 [0.000]	23.689 [0.000]
ARCH(1)	12.830 [0.000]	11.324 [0.001]
ARCH(4)	19.218 [0.000]	20.080 [0.000]
Jarque-Bera	1037.537 [0.000]	716.753 [0.000]

*Note: Standard errors of parameter estimates are reported in parenthesis and p-values of the test statistics are reported in square brackets. log L denotes log likelihood of the estimated model. AIC and BIC are the Akaike and Scharwz's Bayesian information criteria, respectively. Q(k) is the Ljung-Box test of serial correlation for order k in residuals, while $Q^2(k)$ is the McLeod-Li test of serial correlation of order k in squared residuals. ARCH(k) is the LM test of ARCH effect of order k. Q(k), $Q^2(k)$, and ARCH(k) have Chi-Square distribution with k degrees of freedom. Jarque-Bera is the test of normality, which has Chi-Square distribution with 2 degrees of freedom. Statistics for Band-TAR model is computed for the combined residuals. *, **, and *** denote significance at 10%, 5%, and 1%, respectively*

4.2 Point Forecast Results

Out-of-sample point forecast results for the Band-TAR model is reported in Table 2. Columns 2 and 6 represent the root MSFE (RMSFE) of the linear AR model for both the real, rand-dollar and rand-pound exchange rates, hereon referred to as the United States and United Kingdom respectively. Columns 3 and 7 report the ratio of the Band-TAR model RMSFE to the linear AR model RMSFE, i.e. the relative RMSFE. The relative RMSFE is close to unity at very short and very long horizons for the United States (US), while for the United Kingdom(UK), relative RMSFE is greater than unity at all horizons. Results using the M-DM test show that we cannot reject the null hypothesis that the Band-TAR model RMSFE is not less than the linear AR model MSFE. Thus, there is no significant support for the Band-TAR model over the linear AR model for both US and UK real exchange rate forecasts. Hypothesis tests are based on p-values of the Student's t distribution and according to Rapach and Wohar (2006) one should be cautious about making inferences on the Student's t distribution for the M-DM statistics. McCracken (2004) shows that the Diebold and Mariano (1995) statistic has a non-standard limiting distribution when comparing forecasts from two nested linear models when $h = 1$. When comparing forecasts from nested linear models, Clark and McCracken (2004) recommend that a bootstrap procedure should be used to calculate critical values when $h \geq 2$ as the Diebold and Mariano (1995) statistics has a non-standard limiting distribution that is not free of nuisance parameters.

Because the threshold, c from Equation 1 approaches zero in the Band-TAR model, the Band-TAR and linear AR models are nested. Similar to Rapach and Wohar (2006), we make use of a parametric bootstrap procedure in order to generate critical values for the M-DM statistics[10].

Bootstrapped p-values are shown in curly brackets in Tables 2 and 3. When evaluating the two models based on the parametric bootstrapped p-values, the M-DM statistics remains insignificant for both countries' exchange rates. Columns 5 and 9 in Table 2 show results for the MW-DM statistics,[11] as well as the student's t distribution p-values in square brackets and bootstrapped p-values in curly brackets. Again we find that the statistics for both countries' exchange rates are insignificant and find no support for the Band-TAR model over the linear AR model.

[10] Assuming that under the null hypothesis, the data follows a AR(1) process we follow Rapach and Wohar (2006) and simulate a large number of pseudo-samples and compute the M-DM statistic for each pseudo-sample. The bootstrapped p-values are the proportion of the M-DM statistics corresponding to the pseudo-samples greater than the M-DM statistics corresponding to the original sample.

[11] MW-DM statistics place greater weight on forecasting real exchange rates values farther out in the tails of the unconditional distribution.

Table 2: Out-of-sample point forecast evaluation, linear AR and Band-TAR models

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
h[a]	**AR[b]**	**ESTAR/ AR[c]**	**M-DM[d]**	**MW-DM[e]**	**AR[b]**	**ESTAR/ AR[c]**	**M-DM[d]**	**MW-DM[e]**
United States, 1995:01 -2012:07 out of sample period					United Kingdom, 1995:01 -2012:07 out of sample period			
1	0.017	1.067	-2.373 [0.991] {0.732}	-2.409 [0.992] {0.678}	0.03	1.777	-8.500 [1.000] {0.966}	-8.497 [1.000] {0.964}
2	0.029	1.081	-1.44 [0.924] {0.656}	-1.483 [0.930] {0.590}	0.05	1.943	-5.704 [1.000] {0.954}	-5.701 [1.000] {0.952}
3	0.037	1.096	-1.384 [0.916] {0.618}	-1.437 [0.924] {0.578}	0.07	2.058	-5.428 [1.000] {0.944}	-5.409 [1.000] {0.938}
6	0.056	1.125	-1.287 [0.900] {0.598}	-1.345 [0.910] {0.554}	0.1	2.051	-4.69 [1.000] {0.924}	-4.652 [1.000] {0.918}
9	0.070	1.129	-1.121 [0.868] {0.582}	-1.181 [0.881] {0.564}	0.11	1.897	-4.278 [1.000] {0.900}	-4.244 [1.000] {0.900}
12	0.081	1.109	-0.970 [0.833] {0.574}	-1.037 [0.850] {0.548}	0.13	1.731	-4.078 [1.000] {0.900}	-4.055 [1.000] {0.904}
15	0.090	1.092	-0.876 [0.809] {0.564}	-0.955 [0.830] {0.552}	0.13	1.619	-4.052 [1.000] {0.906}	-4.04 [1.000] {0.910}
18	0.097	1.079	-0.839 [0.799] {0.564}	-0.934 [0.824] {0.556}	0.14	1.534	-4.000 [1.000] {0.900}	-4.002 [1.000] {0.910}
21	0.104	1.066	-0.794 [0.786] {0.562}	-0.930 [0.823] {0.564}	0.15	1.470	-4.049 [1.000] {0.894}	-4.051 [1.000] {0.906}
24	0.110	1.051	-0.731 [0.767] {0.552}	-0.896 [0.814] {0.552}	0.15	1.408	-4.096 [1.000] {0.894}	-4.099 [1.000] {0.914}

Notes: p-values using Student's t distribution are reported in square brackets; bootstrapped p-values are reported in curly brackets

a) Forecast horizon (in months).

b) Linear AR model RMSFE.

c) Ratio of the Band-TAR model RMSFE to the linear AR model RMSFE.

d) Modified Diebold and Mariano (1995) test statistic for the null hypothesis that the linear AR model MSFE equals the Band-TAR model. MSFE against the alternative hypothesis that the linear AR model MSFE is greater than the Band-TAR model MSFE.

e) Modified weighted Diebold and Mariano (1995) test statistic for the null hypothesis that the linear AR model weighted MSFE equals the Band-TAR model weighted MSFE against the alternative hypothesis that the linear AR model weighted MSFE is greater than the Band-TAR model weighted MSFE.

In Table 3, we show the out-of sample point forecast results for the ESTAR and linear AR(1) models. For both the rand-dollar and rand-pound exchange rates, the ESTAR models' RMSFE is larger than the linear AR models' RMSFE at all periods. The ESTAR models' RMSFE for the United States is only 1% (3%) less than the linear AR models' RMSFE at the 21-month (24-month) horizon.

Similarly, for the United Kingdom the RMSFE for the ESTAR model was larger than that of the linear AR model at all horizons. Furthermore, the M-DM and MW-DM bootstrapped p-values for both the United States and United Kingdom were insignificant at all periods. We can therefore conclude that for both countries, there is no support for the ESTAR model over the linear AR model. Thus far, evidence is given that the Band-TAR model, as well as the ESTAR model, fails to outperform the linear AR model in out-of-sample point forecasting performance.

Table 3: Out-of-sample point forecast evaluation linear AR and ESTAR models

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
h[a]	**AR[b]**	**Band-TAR/ AR[c]**	**M-DM[d]**	**MW-DM[e]**	**AR[b]**	**Band-TAR/ AR[c]**	**M-DM[d]**	**MW-DM[e]**
United States, 1995:01 -2012:07 out of sample period					United Kingdom, 1995:01 -2012:07 out of sample period			
1	0.016	1.326	-3.225[0.999] {0.835}	-3.227[0.999] {0.830}	0.016	2.116	-6.727[1.00] {0.970}	-6.712[1.00] {0.970}
2	0.027	1.292	-1.953[0.974] {0.790}	-1.964[0.975] {0.765}	0.025	2.095	-4.629[1.00] {0.975}	-4.617[1.00] {0.970}
3	0.034	1.274	-1.745[0.959] {0.745}	-1.765[0.960] {0.760}	0.032	2.106	-4.560[1.00] {0.980}	-4.541[1.00] {0.980}
6	0.050	1.199	-1.334[0.909] {0.700}	-1.379[0.915] {0.715}	0.047	1.978	-4.322[1.00] {0.980}	-4.277[1.00] {0.980}
9	0.062	1.141	-1.030[0.848] {0.665}	-1.085[0.860] {0.690}	0.060	1.834	-4.199[1.00] {0.985}	-4.144[1.00] {0.990}
12	0.073	1.079	-0.703[0.759] {0.615}	-0.777[0.781] {0.655}	0.073	1.697	-4.188[1.00] {0.985}	-4.130[1.00] {0.990}
15	0.083	1.039	-0.422[0.663] {0.545}	-0.519[0.698] {0.630}	0.083	1.613	-4.248[1.00] {0.985}	-4.192[1.00] {0.985}
18	0.090	1.012	-0.167[0.566] {0.510}	-0.293[0.615] {0.600}	0.092	1.805	-4.284[1.00] {0.975}	-4.241[1.00] {0.980}
21	0.097	0.988	0.203[0.419] {0.435}	0.020[0.492] {0.545}	0.100	1.769	-4.395[1.00] {0.975}	-4.344[1.00] {0.980}
24	0.104	0.970	0.705[0.241] {0.305}	0.476[0.317] {0.430}	0.108	1.713	-4.545[1.00] {0.970}	-4.487[1.00] {0.990}

Notes: p-values using Student's t distribution are reported in square brackets; bootstrapped p-values are reported in curly brackets

a) Forecast horizon (in months).

b) Linear AR model RMSFE.

c) Ratio of the ESTAR model RMSFE to the linear AR model RMSFE.

d) Modified Diebold and Mariano (1995) test statistic for the null hypothesis that the linear AR model MSFE equals the ESTAR model MSFE against the alternative hypothesis that the linear AR model MSFE is greater than the ESTAR model MSFE.

e) Modified weighted Diebold and Mariano (1995) test statistic for the null hypothesis that the linear AR model weighted MSFE equals the ESTAR model weighted MSFE against the alternative hypothesis that the linear AR model weighted MSFE is greater than the ESTAR model weighted MSFE.

4.3 Interval forecasts

We evaluate interval forecasts for both the Band-TAR and the linear AR model in Table 4, by evaluating the Pearson χ^2 statistics for $h = 1 - 3$. Following Wallis (2003), we aim to see if we gain more from inter-quartile interval forecasts, as opposed to point forecasts when comparing non-linear models to the linear AR model. Our inter-quartile intervals are at 0.25 and 0.75. Columns 4 and 6 show the correct unconditional coverage (UC) and the correct conditional coverage (CC). For the rand-dollar exchange rate (United States), we fail to reject the unconditional coverage for the linear AR model at all horizons and for the Band-TAR model at horizon 3. Conditional coverage for the US is rejected at all horizons except for the linear AR model at horizons 1. For the rand-pound exchange rate (United Kingdom), we fail to reject unconditional coverage for the linear AR at all three horizons as well as the conditional coverage at horizons one and two. We reject conditional coverage at horizon 1-3 (3) for the Band-TAR (linear AR) model and also unconditional coverage for the Band-TAR model at horizons 1-3. For the US, we reject independence for the linear AR model at horizon two and three as well as the Band-TAR model at horizons 1-3; while for the UK we reject independence for the linear AR model at horizon 3 and for the Band-TAR model at horizons 1-3. The results in Table 4 do not provide support for the Band-TAR model over the linear AR model.

Similarly, Table 5 shows the evaluation of the inter-quartile interval forecasts for the ESTAR and linear AR(1) models. The results show that for the US, the correct unconditional coverage for both the linear AR and the ESTAR models cannot be rejected at any horizon. We fail to reject the correct conditional coverage for the linear AR at horizon one, however we can reject at $h = 2$ and $h = 3$. For the ESTAR model, we reject the correct conditional coverage at all horizons. For the UK, we fail to reject the correct unconditional coverage for the linear AR at all horizons. However, we reject correct unconditional coverage for the ESTAR model at horizons 1-3. We also fail to reject the correct conditional coverage for the linear AR model at horizon 1 and 2, however we reject the null hypothesis at horizon 1-3 for the ESTAR model. For the US, we reject independence at all horizons for both the linear AR and ESTAR model, except at horizon one of the linear AR model. In the case of the UK, we fail to reject independence for the linear AR model at horizon one and two but reject independence for the ESTAR model at all horizons. The results in Table 5 do not provide support for the ESTAR model over the linear AR model.

Table 4: South African real exchange rate with respect to the United States dollar and Great Britain pound

Out-of-sample interval forecast evaluation, linear AR and Band-TAR models

(1)	(2)	(3)	(4)	(5)	(6)
Model	**h[a]**	**0.10/h**	χ^2_{UC}[b]	χ^2_{IND}[c]	χ^2_{CC}[d]
United States, 1995:01 -2012:07 out of sample period					
Linear AR	1	0.10	0.38 [0.69]	4.21 [0.11]	4.51[0.11]
Linear AR	2	0.05	0.47 [0.56], 0.47 [0.56]	**13.89 [0.00], 13.89 [0.00]**	**13.89 [0.00], 13.89 [0.00]**
Linear AR	3	0.033	2.80 [0.12], 0.23 [0.72]	**21.81 [0.00], 21.81 [0.00]**	**22.06 [0.00], 22.06 [0.00]**
			1.75 [0.23]	**20.90 [0.00]**	**21.27 [0.00]**
Band-TAR	1	0.10	**20.02 [0.00]**	**8.58 [0.00]**	**27.28[0.00]**
Band-TAR	2	0.05	**4.20 [0.05], 5.95 [0.02]**	**21.84 [0.00], 21.84 [0.00]**	**22.33 [0.00], 22.33 [0.00]**
Band-TAR	3	0.033	0.51 [0.55], 2.05 [0.15]	**17.51 [0.00], 17.51 [0.00]**	**17.78 [0.00], 17.78 [0.00]**
			5.23[0.03]	**16.66[0.00]**	**17.06[0.00]**
United Kingdom, 1995:01 -2012:07 out of sample period					
Linear AR	1	0.10	2.09 [0.20]	0.71 [0.67]	2.61[0.27]
Linear AR	2	0.05	0.01 [1.00], 0.24 [0.70]	2.42 [0.17], 2.42 [0.17]	2.57 [0.29], 2.57 [0.29]
Linear AR	3	0.033	0.51 [0.48], 0.06 [0.91]	**10.55 [0.00], 10.55 [0.00]**	**10.66 [0.00], 10.66 [0.00]**
			0.13 [0.81]	**9.95 [0.00]**	**10.00 [0.00]**
Band-TAR	1	0.10	**26.66 [0.00]**	**34.99 [0.00]**	**57.91[0.00]**
Band-TAR	2	0.05	**28.81 [0.00], 24.77 [0.00]**	**15.51 [0.00], 15.51 [0.00]**	**41.17 [0.00], 41.17 [0.00]**
Band-TAR	3	0.033	**27.66 [0.00], 32.91 [0.00]**	**20.34 [0.00], 20.34 [0.00]**	**35.89 [0.00], 35.89 [0.00]**
			29.35[0.00]	**19.90[0.00]**	**34.92[0.00]**

Notes: Statistics are reported for each of the h subgroups; the exact p-value is reported in brackets; bold statistic indicate significance at the 0.10 /h level according to the exact p-value; 0.00 indicates <0.005.

a) Forecast horizon (in months).

b) Pearson χ^2 test statistic for the null hypothesis that the prediction intervals have correct unconditional coverage.

c) Pearson χ^2 test statistic for the null hypothesis that the hits relating to the prediction intervals are independent.

d) Pearson χ^2 test statistic for the null hypothesis that the prediction intervals have correct conditional coverage.

Table 5: Out-of-sample interval forecast evaluation, linear AR and ESTAR models

(1)	(2)	(3)	(4)	(5)	(6)
Model	**h**	**0.10/h**	χ^2_{UC}	χ^2_{IND}	χ^2_{CC}
United States, 1995:01 -2012:07 out of sample period					
Linear AR	1	0.1	0.38[0.69]	4.21[0.11]	4.51[0.11]
Linear AR	2	0.05	0.47 [0.56], 0.47 [0.56]	**13.89 [0.00], 13.89 [0.00]**	**13.89 [0.00], 13.89 [0.00]**
Linear AR	3	0.03	2.80 [0.12], 0.23 [0.72]	**21.81 [0.00], 21.81 [0.00]**	**22.06 [0.00], 22.06 [0.00]**
			1.75 [0.23]	**20.90 [0.00]**	**21.27 [0.00]**
ESTAR	1	0.1	1.71[0.24]	**19.92[0.00]**	**21.65 [0.00]**
ESTAR	2	0.05	2.75 [0.12], 0.47 [0.56]	**22.13 [0.00], 22.13 [0.00]**	**22.17 [0.00], 22.17 [0.00]**
ESTAR	3	0.03	0.91 [0.40], 0.00 [1.00]	**21.81 [0.00], 21.81 [0.00]**	**22.06 [0.00], 22.06 [0.00]**
			1.75 [0.23]	**20.90 [0.00]**	**21.27 [0.00]**
United Kingdom, 1995:01 -2012:07 out of sample period					
Linear AR	1	0.1	2.09[0.20]	0.71 [0.67]	2.61 [0.27]
Linear AR	2	0.05	0.01 [1.00], 0.24 [0.70]	2.42 [0.17], 2.42 [0.17]	2.57 [0.29], 2.57 [0.29]
Linear AR	3	0.03	0.51 [0.48], 0.06 [0.90]	**10.55[0.00], 10.55 [0.00]**	**10.66 [0.00], 10.66 [0.00]**
			0.13 [0.81]	**10.55[0.00]**	**10.00[0.00]**
ESTAR	1	0.1	**40.99 [0.00]**	**23.29 [0.00]**	**60.70 [0.00]**
ESTAR	2	0.05	**33.15 [0.00], 30.94 [0.00]**	**12.60 [0.00], 12.60 [0.00]**	**43.02 [0.00], 43.02 [0.00]**
ESTAR	3	0.03	**35.71 [0.00], 30.23 [0.00]**	**21.83 [0.00], 21.82 [0.00]**	**35.39 [0.00], 35.39 [0.00]**
			22.04 [0.00]	**21.36 [0.00]**	**34.43 [0.00]**

Notes: Statistics are reported for each of the h subgroups; the exact p-value is reported in brackets; bold statistic indicate significance at the 0.10 /h level according to the exact p-value; 0.00 indicates <0.005.

a) Forecast horizon (in months).

b) Pearson χ^2 test statistic for the null hypothesis that the prediction intervals have correct unconditional coverage.

c) Pearson χ^2 test statistic for the null hypothesis that the hits relating to the prediction intervals are independent.

d) Pearson χ^2 test statistic for the null hypothesis that the prediction intervals have correct conditional coverage.

4.4 Density forecasts

Results from the density forecast evaluations for the Band-TAR and linear AR(1) model is shown in Table 6. The DH statistic which tests for normality is rejected in the linear AR and the Band-TAR models for both South African rand-US dollar and rand-British pound exchange rates. Similarly, the independent PITs for $k = 2$ is rejected at all horizons for both currencies and for both the linear AR and the Band-TAR models. The DH statistic (column 5) is also strongly rejected for both models and for both the US and UK. The LB statistic $(k = 4)$ in column 9 is also strongly rejected for both models and for both countries, except for the Band-TAR at horizon 2 and 3. The KS statistic is significant at horizon 1 for both models and for both the US and UK. We therefore conclude that there is little support for the Band-TAR model over the linear AR model in terms of density forecasts of real exchange rates.

In Table 7, we show the results for the density forecast evaluation of the ESTAR and AR models. We reject the null hypothesis of independence using the LB statistic $(k = 2)$ for both the ESTAR and the AR models for both the US and UK. For $k = 3$ the LB statistic is not rejected for the ESTAR model for both real exchange rates at horizon 2 and 3. For the UK, the ESTAR is also not rejected for $k = 4$ at all horizons. This means that there are deficiencies in both the ESTAR and linear AR models' specifications; however it seems that the ESTAR has fewer deficiencies in longer horizons. The KS statistic does not reject uniformity for any of the models at horizons 2 and 3 for the US or UK. The DH statistic is significant at all horizons for both the ESTAR and the linear AR models for the US and UK. Therefore, we can conclude that there is little support for the ESTAR model over the linear AR model in terms of density forecasts of real exchange rates.

Table 6: Out-of-sample density forecast evaluation, linear AR and Band-TAR models

	(2)	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**
Model	**h**	**0.10/h**	**KS**	**DH**	**LB,k=1**	**LB,k=2**	**LB,k=3**	**LB,k=4**
United States, 1995:01 -2012:07 out of sample period								
Linear AR	1	0.1	**0.11**	**36.53**	**19.91**	**85.22**	**23.05**	**47.98**
Linear AR	2	0.05	0.18, 0.15	**13.39, 10.26**	0.79, 1.05	**27.78, 36.99**	0.76, 1.62	**11.81, 14.55**
Linear AR	3	0.0333	0.17, 0.17	**16.52, 6.26**	0.25, 1.36	**19.12, 17.89**	1.762, 3.82	**6.56, 11.36**
			0.19	5.605	0.183	**23.907**	0.039	**11.916**
Band-TAR	1	0.1	**0.18**	**107.65**	**21.1**	**70.3**	**14.65**	**29.40**
Band-TAR	2	0.05	0.21, 0.20	**200, 104**	1.38, 3.59	**21.74, 26.36**	0.02, 1.72	3.71, 3.85
Band-TAR	3	0.0333	0.25, 0.22	**67.16, 227**	1.24, 3.59	**15.66, 12.56**	0.80, 0.70	2.23, 1.81
			0.25	**87.46**	0.570	**9.38**	0.02	0.82
United Kingdom, 1995:01 -2012:07 out of sample period								
Linear AR	1	0.1	**0.14**	**26.93**	**10.92**	**74.84**	**8.85**	**30.72**
Linear AR	2	0.05	0.20, 0.16	**9.50,** 5.55	0.01, 0.01	**25.31, 25.08**	0.44, 0.29	**7.56, 7.14**
Linear AR	3	0.0333	0.21, 0.23	**9.47,** 5.22	0.31, 0.02	**20.94, 21.80**	0.47, **4.63**	**10.11, 14.93**
			0.20	1.75	0.26	**30.00**	0.09	**12.13**
Band-TAR	1	0.1	**0.41**	**376**	**67.50**	**39.52**	**11.59**	**4.18**
Band-TAR	2	0.05	0.54, 0.54	**127, 126**	**25.57, 23.89**	**12.51, 13.27**	2.89, 3.37	0.71, 1.23
Band-TAR	3	0.0333	0.61, 0.60	**56.30, 60**	**14.39, 18.59**	**4.18, 6.44**	1.34, 1.68	0.56, 0.60
			0.61	**56.78**	**16.55**	**11.94**	**4.52**	1.83

Notes: Statistics are reported for each of the h subgroups; bold statistic indicate significance at the 0.10 /h level; 0.00 indicates <0.005.

a) Forecast horizon (in months).

b) Kolmogorov–Smirnov test statistic for the null hypothesis that $z_t \sim U(0,1)$.

c)Doornik and Hansen (1994) test statistic for the null hypothesis that $z_t^* \sim N(0,1)$.

d)Ljung–Box test statistic for the null hypothesis of no first-order autocorrelation in $(z_t - \bar{z})^2, k = 1, \dots, 4$.

Table 7: Out-of-sample density forecast evaluation, linear AR and ESTAR models

	(2)	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**
Model	**h**	**0.10/h**	**KS**	**DH**	**LB,k=1**	**LB,k=2**	**LB,k=3**	**LB,k=4**
United States, 1995:01 -2012:07 out of sample period								
Linear AR	1	0.1	**0.11**	**36.53**	**19.91**	**85.22**	**23.05**	**47.98**
Linear AR	2	0.05	0.18, 0.15	**13.39, 10.28**	0.79, 1.05	**27.78, 36.99**	0.76, 1.62	**11.82, 14.55**
Linear AR	3	0.03	0.17, 0.17	**16.52,** 6.26	0.25, 1.36	**19.12, 17.89**	1.76, 3.82	**6.56, 11.36**
			0.19	5.6	0.18	**23.9**	0.04	**11.92**
ESTAR	1	0.1	**0.15**	**399**	**33.74**	**92.54**	**30.54**	**44.55**
ESTAR	2	0.05	0.21, 0.20	**156, 145**	**4.66, 4.50**	**25.52, 34.35**	1.87, 1.60	**6.09, 5.63**
ESTAR	3	0.03	0.27, 0.23	**69.34, 108**	2.48, 4.17	**17.68, 18.23, 18.69**	1.97, 3.07	3.87, **4.76**
			0.25	**93.21**	2.74	**18.69**	0.38	4.13
United Kingdom, 1995:01 -2012:07 out of sample period								
Linear AR	1	0.1	**0.141**	**26.93**	**10.92**	**74.84**	**8.85**	**30.72**
Linear AR	2	0.05	0.20, 0.16	**9.50,** 5.55	0.01, 0.01	**25.31, 25.10**	0.44, 0.29	**7.56, 7.14**
Linear AR	3	0.033	0.21, 0.23	**9.47,** 5.22	0.31, 0.02	**20.94, 21.78**	0.47, **4.63**	**10.11, 14.93**
			0.20	1.75	0.26	**30.00**	0.09	**12.13**
ESTAR	1	0.1	**0.45**	**285**	**62.92**	**31.07**	**6.60**	1.65
ESTAR	2	0.05	0.56, 0.56	**78.48, 73.66**	**21.39, 17.91**	**8.15, 10.27**	1.33, 2.27	0.26, 0.65
ESTAR	3	0.033	0.61, 0.61	**37.38, 46.18**	**10.56, 13.64**	2.54, 4.37	0.52, 1.37	0.16, 0.59
			0.64	**38.70**	**10.61**	**7.22**	1.51	0.36

Notes: Statistics are reported for each of the h subgroups; bold statistic indicate significance at the 0.10 / h level; 0.00 indicates <0.005.

a) Forecast horizon (in months).

b) Kolmogorov–Smirnov test statistic for the null hypothesis that $z_t \sim U(0,1)$.

c)Doornik and Hansen (1994) test statistic for the null hypothesis that $z_t^* \sim N(0,1)$.

d)Ljung–Box test statistic for the null hypothesis of no first-order autocorrelation in $(z_t - \bar{z})^2, k = 1, \dots, 4$.

4.5 Robustness Check

We consider the effect on results reported in Tables 2-7 if we relax the assumption of normally distributed error terms for both the linear and non-linear AR models. Instead of assuming normality we bootstrap the in-sample errors and generate forecasts for the models. Unlike Rapach and Wohar (2006) who report similar results for both models, we obtain the following interesting results for the Band-TAR and ESTAR models.[12]

4.5.1 Band-TAR

The point forecasts of the real exchange rate between the South African rand and the Great Britain Pound (hereon referred to as United Kingdom) are similar to those reported in Table 2. However the real exchange rate with respect to the United States dollar (hereon referred to as United States) shows evidence to support the Band-TAR model over the linear AR model in very long horizons. According to the M-DM and MW-DM statistics, based on p-values of the Student's t distribution, the Band-TARmodel produces superior forecasts in terms of the MSFE and weighted MSFE criterion for periods 21-24.

In our previous results we found some evidence to support linear AR models over Band-TAR models when doing interval forecasts for the United States, which was made clear in Table 4. However if we relax the assumption of normally distributed errors we find evidence to support Band-TAR models over linear AR models. For the United Kingdom, the case was made to support linear AR models when doing interval forecasts for normally distributed errors. After relaxing the assumption, there is no indication that either model outperforms the other. We report no changes in density forecasts for either the United States or United Kingdom when we bootstrap the in-sample errors and generate forecasts.

4.5.2 ESTAR

Point forecasts for the United States are similar to those reported in Table 3, except that M-DM and MW-DM statistics are significant at the 23'rd and 24'th horizon, indicating that the ESTAR performs better at forecasting real exchange rates at very long horizons. Point forecasts for the United Kingdom are unchanged as compared to Table 3, the linear AR model outperforms the non-linear model in terms of the MSFE and weighted MSFE criterion.

There are no changes in interval and density forecasts for either of the countries after we relax the assumption of normally distributed error terms.

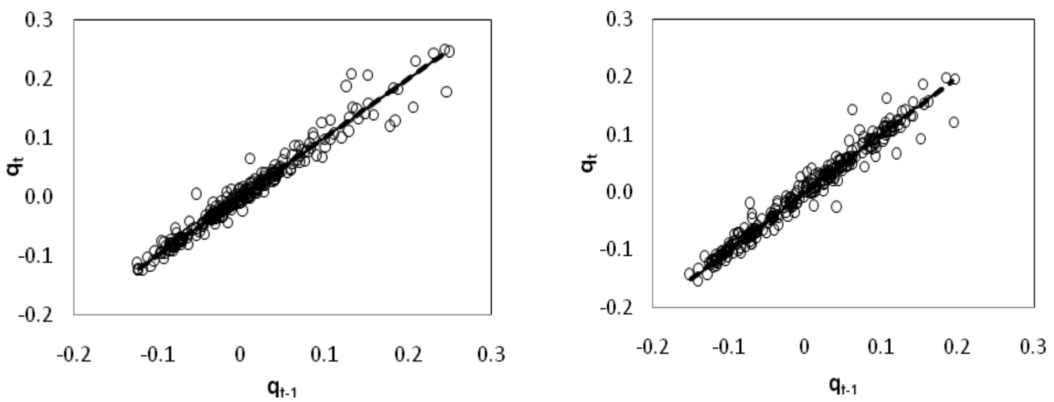
[12] Complete results are not reported to conserve space, they are available upon request.

5. Comparing in-sample conditional densities

If we review the results from Table 2 and 3, it is clear that the MSFE of the point forecasts for the fitted Band-TAR models are very similar to that of the linear AR model. The ESTAR performed better than the Band-TAR model, however the ESTAR had similar forecasting performance as the linear AR model. Diebold and Nason (1990: p318) suggest that this might be because "very slight conditional mean non-linearities might be truly present and be detectable with large datasets, while nevertheless yielding negligible ex ante forecast improvement".

Figure 2: Band-TAR and linear AR(1) scatterplot of real exchange rate (q_t) and lagged exchange rates q_{t-1} in log-level.

A: United States dollar *B: Great Britain pound*



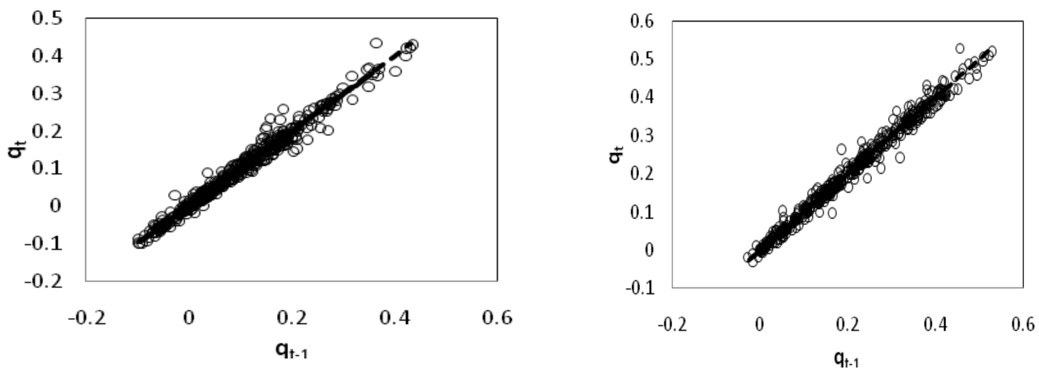
Note: The solid line is the conditional expectations function for the fitted Band-TAR model and the dashed line is the conditional expectations function for the fitted linear AR(1) model.

In order to examine the relevance of this explanation for the specified Band-TAR and ESTAR models, we follow Rapach and Wohar (2006), we graphically compare the conditional expectation functions for q_t given q_{t-1}, related to the fitted Band-TAR and ESTAR models and the linear AR models. This gives us a visual feel for how "close" the fitted linear and nonlinear AR models are in terms of their conditional means. These results are presented in Figure 2, together with a scatter plot of the in-sample data. Figure 2 shows that the conditional expectations function from the fitted Band-TAR and linear AR models are very close to each other, indicating that any non-linearities in the conditional means appear to be very small. From Figure 3 we see that the conditional expectations corresponding to the fitted ESTAR models and the linear AR models are also very close to each other.

We can infer from Figure 2 and 3 that there is not much to be gained in terms of forecasting in the short term when using the Band-TAR and ESTAR models, compared to the linear AR models. This seems to be because of a lack of strong non-linearities in the conditional means of the non-linear AR models.

Figure 3: ESTAR and linear AR(1) scatterplot of real exchange rate (q_t) and lagged exchange rates q_{t-1} in log-level.

A: United States dollar *B: Great Britain Pound*



Note: the solid line is the conditional expectations function for the fitted ESTAR model and the dashed line is the conditional expectations function for the fitted linear AR(1) model.

Table 8: In-sample comparison of conditional densities corresponding to fitted non-linear and linear AR models

	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Block bootstrapped Z_tc.v's				**Block bootstrapped $R_t - Z_t$c.v's**		
Country in-sample period	**Model**	Z_t	**10%**	**5%**	**1%**	$(R_t - Z_t)^2$	**10%**	**5%**	**1%**
United States 1970:01-1994:12	Band-TAR	0.012	0.0239	0.0256	0.0283	0.0148	0.0214	0.024	0.0268
United Kingdom 1970:01-1994:12	Band-TAR	-0.0001	0.0046	0.0085	0.0134	-0.0066	0.0029	0.0037	0.0055
United States 1970:01-2012:07	ESTAR	-0.0001	0.0074	0.0093	0.0121	-0.0032	0.0055	0.0076	0.0104
United Kingdom 1970:01-2012:07	ESTAR	-0.0001	0.0047	0.0059	0.0100	-0.0018	0.0044	0.0063	0.0088

Notes: Bold bootstrapped critical value indicate that the statistic is significant according to the bootstrapped critical value (c.v).

a)Corradi and Swanson (2006) test for the null hypothesis that the conditional densities corresponding to the non-linear and linear AR models are equally accurate relative to the true conditional density against the alternative that the conditional density corresponding to the non-linear AR model is more accurate than the conditional density corresponding to the linear AR model.

b)Corradi and Swanson (2006) test for the null hypothesis that the conditional densities corresponding to the non-linear and linear AR models are equally accurate relative to the true conditional density against the alternative that the conditional density corresponding to the non-linear AR model is more accurate than the conditional density corresponding to the linear AR model for values of qt in the upper and lower quartiles of the in-sample observations.

To compare the fitted non-linear and linear AR models formally, we use the Corradi and Swanson (2006) Z_T statistic. Under the null hypothesis the conditional densities corresponding to the fitted linear and non-linear AR models are equally accurate, relative to the true conditional densities corresponding to a linear AR benchmark model. The Z_T is calculated by integrating the minimum and maximum values of the in-sample q_t observations over a fine grid. A second test statistic $R - Z_T$ is also calculated, which integrates over two grids of values, whose limits correspond to the minimum and maximum values of the first and fourth quartiles of the in-sample observations. We can now compare the conditional distribution matching the fitted linear and non-linear AR models on the tails of the in-sample q_t observations distribution. We report these results in Table 8. As proposed by Corradie and Swanson (2006) and used by Rapach and Wohar (2006), we calculate block bootstrapped critical values. According to the Z_T and $R - Z_T$ statistic, we cannot reject the null hypothesis of equal conditional density accuracy for any of the non-linear AR models relative to the linear AR models. This indicates that relative to the linear AR models, the Band-TAR and the ESTAR models are not significantly different in their conditional densities for q_t given q_{t-1}. We can conclude from Table 8 that the fitted non-linear AR models are close to the fitted linear AR models. This indicates that point and density forecasts generated by Band-TAR and ESTAR models do not sufficiently improve on forecasts generated by linear AR models in the short-run.

6. Conclusion

This paper evaluates the out-of-sample forecasting performance of non-linear models against linear models for the South African rand against two main currencies, the United States dollar and the British pound, where we also adjust for prices. We used monthly data for the period 1970:01 to 2012:07 and estimated non-linear Band-TAR and ESTAR models and compared the outcomes to a linear AR model.

We constructed multi-step point, interval and density forecasts for the non-linear Band-TAR and ESTAR models and linear AR models. This was done to compare the out-of-sample real exchange rate forecasting performance of non-linear models to linear models over a period of twenty four months. Our results showed that there were not significant gains in terms of the out-of-sample forecasting performance of non-linear models compared to linear models, especially in the short-run. These results are true in the case of point, interval and density forecasts.

After we relaxed the assumption of normally distributed errors we found that point forecasts of the real exchange rate with respect to the United States dollar showed evidence to support the Band-TAR model over the linear AR model for periods 21-24. Interval forecasts for the United States show evidence to support Band-TAR models over linear AR models as well. Point forecasts were significant at the 23'rd and 24'th horizon, indicating that the ESTAR performs better at forecasting real exchange rates at very long horizons than the linear AR model.

These results are consistent with findings from a number of studies.[13] A number of reasons were provided by Diebold and Nason (1990, pp.317-318) explaining why nonlinear models may fail to offer sizable forecasting gains relative to linear models. First, "the nonlinearities may be present in even-ordered conditional moments, and therefore are not useful for point prediction". Second, it is not obvious that the in-sample features of nonlinear time series such as structural breaks and outliers will result in improved out-of-sample forecasts of the nonlinear models compared to those from linear models. Third, "very slight conditional-mean nonlinearities might be truly present and be detectable with large datasets, while nevertheless yielding negligible ex ante forecast improvement".[14] In order to examine the relevance of the third explanation for the Obstfeld and Taylor (1997) Band-TAR and Taylor et al. (2001) ESTAR models, we graphically compare the conditional expectation functions related to the fitted Band-TAR and ESTAR models and the linear AR models. When we construct in-sample, one-month-ahead conditional expectation functions and conditional densities, we found that the Band-TAR and ESTAR models were not different from linear AR models. The lack of sizable forecasting gains especially at short horizons provided by the fitted Band-TAR and ESTAR models relative to their linear AR counterparts appears to result from the absence of strong nonlinearities in the conditional means of these nonlinear AR models. Further, the switch variable for instance, could be at longer lags but staying with theory may have possibly prevented us from modeling the non-linearities properly. We thus, draw the conclusion that any non-linearities in monthly real exchange rates for South Africa against the US dollar and against the British pound is too weak for the fitted Band-TAR and ESTAR models to estimate accurately.

[13] See Diebold and Nason (1990), Granger and Terasvirta (1993), Clements and Hendry (2001), Liu and Prodan (2007), Buncic (2009).

[14] According to them, "the *significance* of nonlinearity does not necessarily imply its *economic importance*."

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Appendix 1: Summary statistics of South African exchange rates

Statistics	**rand- dollar**	**rand- pound**
Mean	0.800	0.980
Maximum	1.150	1.301
Minimum	0.616	0.743
Standard deviation	0.103	0.128
Skewness	0.730	0.189
Kurtosis	3.355	2.134
Jarque-Bera	48.098 (0.000)	19.017 (0.000)

Note: p-value for the Jarque-Bera statistics is in parenthesis.