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# The Oil Price and Exchange Rate Relationship Revisited: A time-varying VAR parameter approach

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**Abstract**

The aim of this paper is to study the relationship between the effective exchange rate of the dollar and the oil price dynamics from 1976 to 2013. We explore the links between financial factors (exchange rate, monetary policy, international liquidity) and the oil price volatility. Using a Bayesian time-varying parameter vector auto-regressive estimation we demonstrate that the "historical coincidence" of oil and financial crises can be explained by the specificities of the relationship between these two commodities. The results of this paper are twofold. The US Dollar effective exchange rate elasticity of crude oil prices is not constant across time and remains negative from 1989: a depreciation of the effective exchange rate of the dollar triggers an increase of crude oil prices. This paper also demonstrates the contagion of financial commodities markets development upon the global economy.

JEL: F31;Q43;C32

Keywords: Exchange rate, oil price, TVP-VAR.

## 1. Introduction

### 1.1 Context

Since the beginning of the 1970s, foreign exchange and oil markets both sustained shocks and crises. With the unilateral cancellation of the direct convertibility of the US Dollar to gold on August 15, 1971, the world economy experienced a new financial context with the advent of a floating foreign exchange market. In 1976 the Jamaica Agreement replaced *de facto* the Bretton Woods System based on exchange rate stability. In the oil markets, the progressive take-over by the Organization of the Petroleum Exporting Countries (OPEC), at the beginning of the 1970s, and the first oil shock in 1973-1974 can be considered as the starting point of a new context of oil price volatility compared to the previous decade. As oil is quoted in US dollars (henceforth, USD), the oil prices and the value of the dollar seem to have a mutual influence and it is relevant to study the interaction between those two variables. Several studies have previously investigated the link between the USD and the oil prices.

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**1.2 Literature Review**

Krugman (1980, 1983) analyses the variation of the USD following an oil price increase in a model with three different areas (America, Germany and the OPECmembers). He demonstrates that the final effect on the Dollar exchange rate in the short run heavily depends on the comparison between the US weight in world oil imports and the share of Dollar assets in OPEC's portfolio; while in the long run, the comparison has to be made between the US share in world oil imports and the weight of US goods in OPEC's imports. Golub (1983) divides the world in three areas (America, Europe and the OPEC countries) and two currencies (USD and Deutsche Mark) and focuses on the wealth transfers to the OPEC countries. Under the assumption of inelastic demand of oil from Europe and America, the USD depreciates against the German currency if the OPEC countries have a higher propensity to hold Deutsche Marks than the oil-importing countries. If, as a result of the wealth transfer, there is an excess demand for Deutsche Mark, then the dollar exchange rate will depreciate.

There is no clear consensus in the literature concerning the direction (influence of the dollar on the oil price or *vice versa*) and the sign of the relationship between oil price and the value of the Dollar (e.g., Beckmann and Czudaj, 2013). Regarding the direction of the relationship, some studies focused on the oil price to exchange rate causality. Chen and Chen (2007) investigate the long-term relationship between the real oil prices and the real exchange rate. Using a panel data methodology, they find a cointegration relationship using a set of different markers of crude oil or basket price (Brent, Dubai, West Texas Intermediate (WTI) and world prices), and conclude that the oil prices are "the dominant source of real exchange rate movements". However, Sadorsky (2000) and Krichene (2007) among others find a reverse causality from USD to oil prices. Therefore, the direction of the causality between oil price and exchange rate is not clear cut. Percebois (2009) summarizes the complexity of the relationship by emphasizing the possibility of a bilateral causality through various macroeconomic channels such as: the effect on world demand through local prices, a Chinese effect, a petrodollar recycling effect, and a target revenue effect. Bénassy-Quéré et al. (2007) highlight the existence of a cointegration relationship between the oil price and the Dollar real effective exchange rate for the 1974-2004 period with the causality link running from oil price to the exchange rate. Nevertheless, the authors suggest that the causality link could be reversed in the 2002-2004 period because of the Chinese monetary policy. Benhmad (2012), using wavelet analysis, also emphasized that time scales matter. Causality between real oil price and real effective US Dollar exchange rate returns runs in bidirectional ways in long time horizons, while in short time horizons, causality runs from oil prices to real effective US Dollar exchange rate returns.

Regarding the sign of the relationship, there is also no clear consensus in the empirical literature. Amano and Van Norden (1998a, 1998b) show that the impact of an oil price increase on the exchange rate is heterogeneous across countries: a 10% increase in the oil price leads to a depreciation of both Japanese and German currencies (respectively of 1.7 and 0.9%), when it causes an appreciation of the USD of around 2.4%. Relying on two monetary models (Basic and Composite Models), Lizardo and Mollick (2010) highlight that the dependency to oil could play a major role in the relationship. They split the country panel into two groups (crude oil net importers or net exporters). They demonstrate that an increase in the oil price is followed by an appreciation of the net exporters' currencies relative to the USD, while for the oil importing countries, their currencies tend to decrease relative to the USD. Reboredo et al. (2014) study the relationship between WTI price and the exchange rate of US Dollar against a set of currencies[1], splitting the period of studies into two samples: one before the crisis (July 2008) and one after. The authors find that the cross-correlations between oil price and exchange rates are negative and that the dependence increases after the start of the financial crisis. With an analysis based on impulse responses and forecast error variance decomposition, Akram (2009) focuses on the relationship between oil prices, commodity prices, the USD exchange rate, the global output and interest rates. Building three VAR models, he argues that a positive shock on the oil price is followed by real exchange rate depreciation, while a real exchange rate depreciation leads to higher commodity prices (including oil prices). Creti et al. (2013) using a dynamic conditional correlation (DCC) GARCH methodology investigate the links between 25 commodities and stocks from 2001 to 2011 and conclude that the correlations among those variables are not constant over time and are highly volatile since the financial crisis. Finally Jebabli et al. (2014) using a new time varying parameter VAR (TVP-VAR) model with a stochastic volatility approach for a large set of commodities (crops, livestock, plantation and forestry products) analyse the spill-over effect between financial, food and energy markets. They demonstrate an increase of markets contagion since 2008.

Thus, there is no clear-cut result regarding neither the direction of the relationship between exchange rate and oil price, nor the extent of the elasticity due essentially to different sample period considered in the previous listed studies. In our paper, we analyse the oil-dollar link with an innovative methodology. To improve the comprehension of the oil price-exchange rate relationship, we use time-varying Bayesian VAR to estimate the different parameters of interest. This methodology offers four main advantages in our analysis. First, as a VAR analysis, it takes into account endogeneity among the set of variables and, thus, permits to circumvent the absence of consensus concerning the direction of the relationship. Second, the estimated elasticities, which measure the movement in percentage of the variables of interest following one percent change of one variable, evolve through the whole estimation period, enlarging the scope of the interpretation (in terms of magnitude and periodicity) compared to previous empirical studies. Third,

[1] Respectively the Australian Dollar, the Canadian Dollar, the Japanese Yen, the Mexican Peso, the Euro, the Norwegian Krone and the British Pound.

it permits to model both abrupt break and persistence in the relationship between variables of interest. Indeed, relationship between oil price and exchange rate might be subject to structural break due to different exogenous oil events such as geopolitical events, OPEC decisions, economic or monetary policy changes, etc. It might also be subject to gradual evolution due to adaptative learning behaviour of agents (Primiceri, 2005). Therefore, letting data determine whether the relationship presents a break or is persistent is a valuable future of this methodology. Fourth, the Forecast Error Variance Decomposition (henceforth FEDV) which measures the contribution of different variables in the model to the volatility of the variable of interest is calculated for the entire period of the sample. Up to our knowledge, this paper is the first one that uses the time-varying FEDV for the oil prices-exchange rate analysis and, hence, contributes to a more comprehensive analysis of this subject with regards to time-changing economic environment pattern and monetary policy during the estimation period.

We use monthly data on oil price (Spot market Brent price expressed in US dollars)[2], the gold price (spot price in the London Commodity Exchange measured in US dollars), the effective exchange rate of the Dollar, the HWWI index[3] (Industrial raw materials measured in US dollars) and the dry cargo index as a proxy for the real economic activity. Our monthly data cover the sample period from 1976:07 to 2013:07. Oil and gold prices are extracted from Datastream database (respectively, with code "UKI..C..A" and "UKOILBREN"). The US Dollar effective exchange rate we use is the nominal major currencies Dollar index extracted from the board of governors of the Federal Reserve System (Foreign Exchange Rates - H.10). Finally, the Dry Cargo index is available at Lutz Kilian personal webpage. Except for Dry cargo index, all variables are rendered stationary by taking the first difference of their natural logarithm. The rest of the paper is organized as follows: Section 2 advances arguments why the link between oil prices and macroeconomic and financial variables, more particularly the exchange rate, is time-varying; Section 3 is dedicated to the model specification; Empirical results are provided in Section 4; Section 5 concludes the article.

## 2. Why the link between oil prices and exchange rate has evolved over time?

In this paper, we focus on the link between financial factors, more particularly the exchange rate, and oil prices. Thus, this study is an extension of the recent literature that focuses on the link between oil and macroeconomic since the seminal work of Kilian (2009). One of the major concerns in the literature is the origin of oil price shocks and the debate around the exogeneity (the supply

[2] We use Brent Price in this article instead of WTI or OPEC basket price because Brent helps to price around 70% of the global oil transactions. In addition we consider that WTI has, since the last decade, a very specific history and tends to disconnect from the international oil market.

[3] See Box 1 in Appendix A for a more accurate decomposition of the HWWI Index.

channel) and endogeneity (the demand channel) of oil price dynamics with respect to global macroeconomic activity.[4] There has been considerable evidence in the recent literature that changes in global economic activity are the primary source of oil price dynamics (Barsky and Kilian, 2004; Kilian 2008 a,b, 2009; Alquist et al. 2013; Kilian and Hicks, 2013; Kilian and Murphy 2014, to cite a few). This is in contrast with the traditional view that oil price dynamics are merely explained by exogenous events that affect oil producing countries (Hamilton, 2003). For instance, the literature points out that the sharp oil price increase in 1973-74 was explained by 75% of global demand and 25% of supply flow; the sustained increase in the oil price in the 2000s and the dramatic drop of the oil price in the third quarter of 2008 were the results of sustained global economic growth and world trade collapse, respectively.[5] More importantly, global economic activity has a great influence not only on oil prices but also on commodity prices (Baumeister and Kilian, 2014; Joets et al. 2015). Therefore, in line with recent evidences, we take into account in our analysis the demand channel and the endogeneity of oil price with respect to global economic activity. In this vein, we include in our model the Dry Cargo Index that measures the evolution of global economic activity such as Kilian (2009), Baumeister and Peersman (2013), and Kilian and Murphy (2014).

Indeed, the evidence of the demand channel approach is supported by the close link that exists between oil prices and the economy, and policy debates that it raises. There exists a further consensus in the recent literature that this link is not stable over time. There are several reasons why the relationship between oil prices and macroeconomic and financial variables is likely to be time varying. First, foreign exchange market and the oil markets have registered a strong increase in volatility since 1970s whereas real economic activity encountered a significant decrease in volatility during the great moderation. Second, literature on oil prices such as Kilian (2009) argued that shocks underlying oil price fluctuations come from different sources such as oil supply disruption and physical or precautionary oil demand surge. Thus, given that the different natures of oil price shocks generally occur at different periods, link between oil price and other variables is likely to change over time. Third, the growing financialisation of commodities markets since 2000 and more especially the financial crises of 2007-08 could have change the link between oil prices and other commodity prices (Fattouh et al., 2013). Fourth, the literature also advances different explanations such as change in

4 The exogeneity refers to the conventional wisdom that oil price dynamics are merely driven by exogenous events that affect oil producing countries such as war, OPEC cartel's decision, embargoes, political events in the Middle East, civil unrest, ... In turn, the endogeneity refers to the influence of global economic activity on oil price dynamics.

5 As argued by Kilian and Murphy (2014), only the 1990s oil price episode was explained primarily by supply shock. The episodes of the late 1985 and 2007-08 was advanced by the literature (see Fattouh et al., 2013 for a discussion) as the result of speculative behaviours in the oil market which is also called as "speculative demand." For a discussion of different oil episodes and the role of global demand, supply flow and speculative demand, please refer to Barsky and Kilian (2004).

monetary policy, change in oil intensity of economic activity and the regulation of energy market (Baumeister and Peersman, 2013).

## 3. Model specification

### 3.1 Time-varying parameters

To highlight the influence of the effective exchange rate of the USD on oil prices during the whole estimation period, we use the Bayesian time-varying VAR estimation approach. Starting from the state-space representation of the reduced form of the VAR model, the entire sequence of the time varying regression parameters and their respective variances are generated via forward and backward recursion of Kalman filter, thus using all the information available throughout the entire estimation period. The entire sequences of parameters of interest are estimated by simulating their distribution using Bayesian approach, namely we implement a Gibbs sampler. Details of the estimation procedure are available in the appendix[6].

#### 3.1.1 The model

Consider the following structural VAR representation of a multivariate time series model with both time-varying coefficients, contemporaneous and lagged, and time-varying standard-error of structural innovations:

$$B_t Y_t = d_t + C_{1,t} Y_{t-1} + \cdots + C_{p,t} Y_{t-p} + \Sigma_t \upsilon_t \tag{1}$$

where $Y_t = [Y_{1t}', Y_{2t}', \dots, Y_{nt}']'$ is a vector of *n* endogenous variables, $d_t = [d_{1t}', d_{2t}', \dots, d_{nt}']'$ is a vector of *n* time-varying constants, $C_{pt}$ is the matrix of time-varying lag coefficients of the structural model and $\upsilon_t$ is a vector of structural innovations which is assumed to follow a multivariate normal distribution,

$$\upsilon_t \rightsquigarrow \mathcal{N}(\mathbf{0}, \mathbf{I}_n)$$

Using AIC, BIC and HQ information criterions, we retain one lag for the estimation. Indeed, changes in the relationship between variables in the model can be the result of changes in the contemporaneous relationship $B_t$ between variables,

[6] Our methodology draws on that of Cogley and Sargent (2005), and Primiceri (2005) with some modifications.

changes in the propagation mechanism $C_{pt}$ and changes in the size of the standard error of innovations $\Sigma_t$. Therefore, allowing parameters of interest to vary over time leaves it up to the data to determine the nature and the time-varying evolution of this relationship.

Moreover, we assume that the matrix of time-varying contemporaneous coefficients $B_t$ is lower triangular with ones along its diagonal elements

$$B_t = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ b_{21,t} & 1 & 0 & \cdots & 0 \\ b_{31,t} & b_{32,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ b_{n1,t} & b_{n2,t} & \cdots & b_{nn-1,t} & 1 \end{pmatrix}$$

whereas the matrix of time-varying standard-error $\Sigma_t$ is diagonal.

$$\Sigma_t = \begin{pmatrix} \sigma_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{n,t} \end{pmatrix}$$

For estimation purpose, reduced form representation of the structural model (1) is given by:

$$Y_t = c_t + A_{1,t}Y_{t-1} + \cdots + A_{p,t}Y_{t-p} + \epsilon_t \qquad (2)$$

where $A_{p,t} = B_t^{-1}C_{p,t}$ are the matrices of lag-coefficients, $c_t = B_t^{-1}d_t$ is the vector of constants and $\epsilon_t = B_t^{-1}\Sigma_t\upsilon_t$ is the vector of reduced-form residuals. Following the structure of the contemporaneous coefficients matrix $B_t$ and that of the standard error of the structural innovations matrix $\sum_t$, we can assume that reduced-form residuals have the following structure:

$$\epsilon_t \rightsquigarrow \mathcal{N}(\mathbf{0}, \Omega_t)$$

where $\Omega_t$ is a symmetric and positive definite time-varying variance-covariance matrix of $\epsilon_t$ that verifies the following equality,

$$B_t \Omega_t B_t' = \Sigma_t \Sigma_t' \tag{3}$$

It is worth noting that this structure implies a Cholesky identification scheme restricting the matrix of contemporaneous relationship to be lower triangular. Thus, the order of variables in the reduced-form representation (2) matters[7]. However, after estimating the model, it is possible to choose a decomposition of $\Omega_t$ satisfying $S_t S_t^{'} = \Omega_t$ allowing for richer identification strategies.

Time paths for parameters of interest are assumed to be random walks without drifts[8]. If we denote $\alpha_t$ the column vector that contains stacked columns of matrix $A_t$, $b_t = \left(b_{21,t}\ b_{31,t}\ b_{32,t}\ \cdots\ b_{nn-1,t}\right)'$ the column vector that contains the elements of the matrix of contemporaneous relationship $B_t$, $\sigma_t = \left(\sigma_{1,t} \cdots \sigma_{n,t}\right)'$ the column vector that contains the diagonal elements of the matrix of standard error $\Sigma_t$ and $h_t = ln(\sigma_t)$ the natural logarithm of the standard error, parameters evolve according to:

$$\begin{aligned} \alpha_t &= \alpha_{t-1} + \omega_t \\ b_t &= b_{t-1} + \zeta_t \\ h_t &= h_{t-1} + \eta_t \end{aligned} \tag{5}$$

This random walk specification has two main advantages. First, it permits to model possible abrupt break in the evolution of parameters that might occur during the estimation period. Second, it permits also to model gradual changes in the relationship between variables as a result of an adaptative learning behavior of individuals. Therefore, unlike multivariate models used in the literature that model either structural break or persistence, the TVP-VAR encompasses these two important features of the relationship between variables that might occur at different periods in our sample.

[7] As explained in Primiceri (2005), if one is particularly concerned about this problem, a natural solution is to impose a prior on all plausible orders of the variables and, after estimation, average on the basis of the prior or the posterior of different models.

[8] Even though the dynamics of the parameters can be easily extended to a more general autoregressive process, we assume random walk process in order to focus on possible permanent shifts and to reduce the number of parameters in the estimation procedure.

Moreover, innovations in the reduced-form model are assumed to be jointly distributed

$$\begin{pmatrix} \upsilon_t \\ \omega_t \\ \zeta_t \\ \eta_t \end{pmatrix} \rightsquigarrow \mathcal{N}(\mathbf{0}, V) \text{ with } V = \begin{pmatrix} \mathbf{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & Q & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & S & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & W \end{pmatrix}$$

where the matrix $S$ is block diagonal. That is, we assume that blocks, corresponding to contemporaneous coefficients in each equation, are mutually independent. Each block of $S$ corresponds to the variance-covariance matrix of contemporaneous coefficients of each equation.

## 4. Empirical results

The time-varying VAR parameter approach allows us to study the evolution of the crude oil price elasticity with respect to the effective exchange rate of the dollar, the HWWI index, the Gold prices and the dry cargo index.

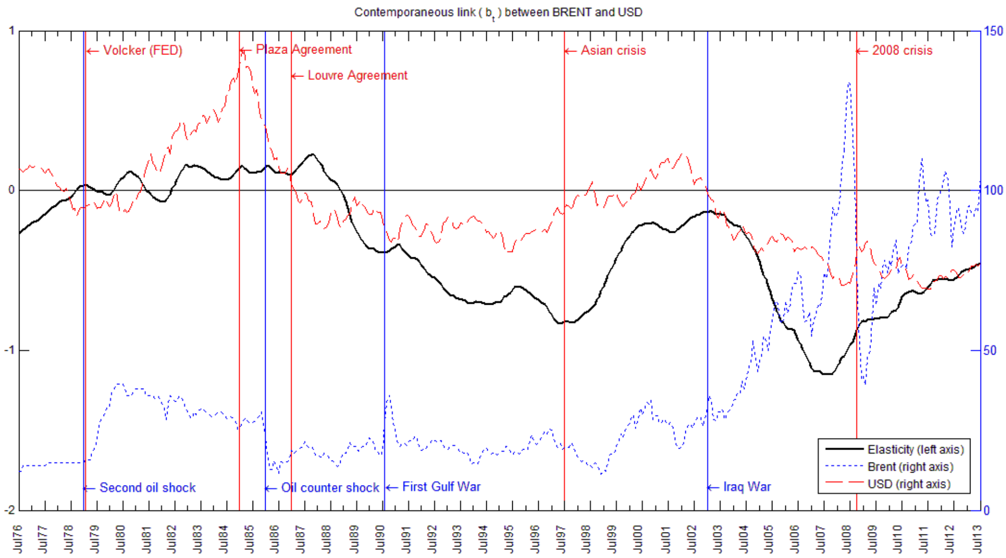
### 4.1 Elasticities

#### 4.1.1 Brent and Effective Exchange rate of the dollar

We report the evolution of the USD elasticity of the oil price in figure 1.[9] Except during the 1979-1989 period where the elasticity is positive (between 0.0 and 0.2), the relation between the effective exchange rate of the dollar and the oil price is negative. It highlights the fact that a depreciation of the effective exchange rate of the dollar leads to an oil price increase. From 1989 to 2013 we then observe a succession of elasticity's cycles: from 1989 to 1997 (from 0.0 to -0.8), from 1997 to mid-2003 (from -0.8 to -0.2), from mid-2003 to June 2008 (from -0.2 to -1.2) and from 2008 to 2013 (from -1.2 to -0.45).

[9] Figures including confidence intervals are reported in Appendix C (Figure 6).

Figure 1: USD Elasticity of Brent




*Source: Authors calculation*

The first period (1979-1989) is characterized by an increase of the crude oil prices in the wake of the second world oil shock. Moreover in August 1979 in the US, Paul Volcker became the chairman of the Federal Reserve Bank and decided to raise the federal fund rate from around 11% in 1979 to a peak at 20% in 1981. This policy led to an increase of about 45% of the effective exchange rate of the dollar between 1979 and 1985. Thereafter the Plaza agreement in 1985 between France, Japan, the United Kingdom, the US, and West Germany led to a sharp depreciation of the effective exchange rate of the dollar between 1985 and 1989. At the same time in the oil market Saudi-Arabia, which reduced its oil production from 9.9 million barrels per day (mb/d) to 3.3 mb/d, failed to maintain oil prices at a reasonable prices and changed its production policy at the beginning of 1986 in order to maintain its market share. This new strategy led to the oil prices collapsed of 1986 to less than 10 USD per barrel and called “Oil counter shock.”

During the second period (1989-2013) the relation between the effective exchange rate of the dollar and the oil prices is negative with an elasticity range from 0.0 to -1.2. These two decades are characterized by two key trends. Except during the Gulf War (1991) and the Asian crisis (1997-1999) the oil markets were relatively stable during the 1990s. However the effective exchange rate of the dollar was quite volatile with a strong appreciation process of the USD between 1996 and 2002.

Two important processes can explain the evolution of the elasticity during the 2000s. In 2001, after ten year of economic expansion, the US GDP registered a sharp contraction in the wake of the 2000-01 new economy financial crash. The Federal Reserve (FED) decided to launch an aggressive easing monetary policy with a sharp decline of the federal fund rate (from 6.5% in January 2001 to 1% in June 2003). This policy helped the effective exchange rate of the dollar to heavily depreciate. During the same time, fuelled by the economic development of the BRICs (Brazil, Russia, India, China and South-Africa) economies and more especially China, the world entered a new era with a sharp increase of oil prices (from 40 USD per barrel in 2004 to 70 USD in 2007 and the peak of around 145 USD in July 2008). It explains the elasticity path from 2001 to 2008 (-0.2 to -1.2).

The relationship between the oil prices and the USD tends to reinforce during the depreciation periods, whereas it weakens during the periods of appreciation. This illustrates an "asymmetric response" between the oil price and the exchange rate depending on the global economic context. Crude oil price behaviour could possibly explain this asymmetry as, other things being equal, a depreciation of the exchange rate (and then a decrease in the value of the oil production) should be followed by an increase of the crude oil price, in order to maintain oil producers revenues. Some studies (Alhajji & Huettner, 2000 among others) analyse this policy as a "target revenue strategy." But this theory assumes a producer ability for being price maker in the oil market whatever the market conditions. This hypothesis remains uncertain from 1973 to 2009 (Brémond et al., 2012). A more plausible explanation comes from the recent literature that support the endogeneity of oil prices with respect to economic activity (Kilian, 2009; Kilian and Hicks, 2013; Kilian and Murphy 2014, to cite a few). Given that oil prices are labeled in USD, a depreciation of the Dollar leads to a decrease in the value of oil in terms of other currencies. It has the result of stimulating demand for oil, namely from emerging market, and hence, exert an upward pressure on oil prices. On the contrary, an appreciation of the Dollar does not trigger any decrease of the crude oil price. It is worth noting that the relationship appears to be weaker during these periods.

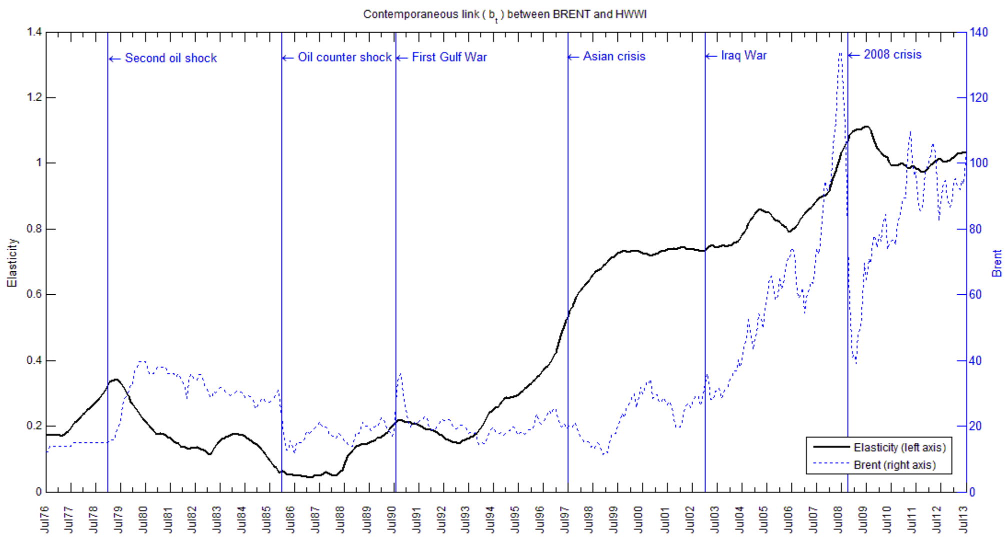
### 4.1.2 Elasticity between HWWI and Brent

Turning to the HWWI raw materials index, figure 2 shows that the elasticity with the Brent is positive and is reinforced in our sample (from 0.2 to 1.1). It illustrates the common trend observed from 2001 to 2008 in the non-energy commodities and in the energy markets.

Indeed although the timing and the magnitude have been different in the various commodities markets (energy, non-ferrous metals, agricultural raw materials and beverages), the price increase that began in late 2001 in the non-ferrous metals had spread into all commodities markets by 2004-2005 in the context of steady world economic growth. This result highlights the possible existence of spillover effect between energy and industrial raw material prices.

The financialisation process of the commodities markets observed since the beginning of 2000 in the wake of the Commodity Future Modernisation Act (CFMA) could explain the reinforcement of the elasticity since the turn of the century. The new financial tools such as the Exchange Traded Fund (ETF) experimented by the financial sector could help to understand this result. Masters (2008) in a Testimony and in different reports for the United States Senate launched the controversial debate about the role of Index commodities funds as drivers of speculation in all the commodities markets. The new hedging strategies and the speculative trading (Hache and Lantz, 2013) that can occur between different market places (New York and London for example) with different commodities (energy, non-ferrous metals and beverages) could also explain this result.

Figure 2: HWWI Elasticity of Brent

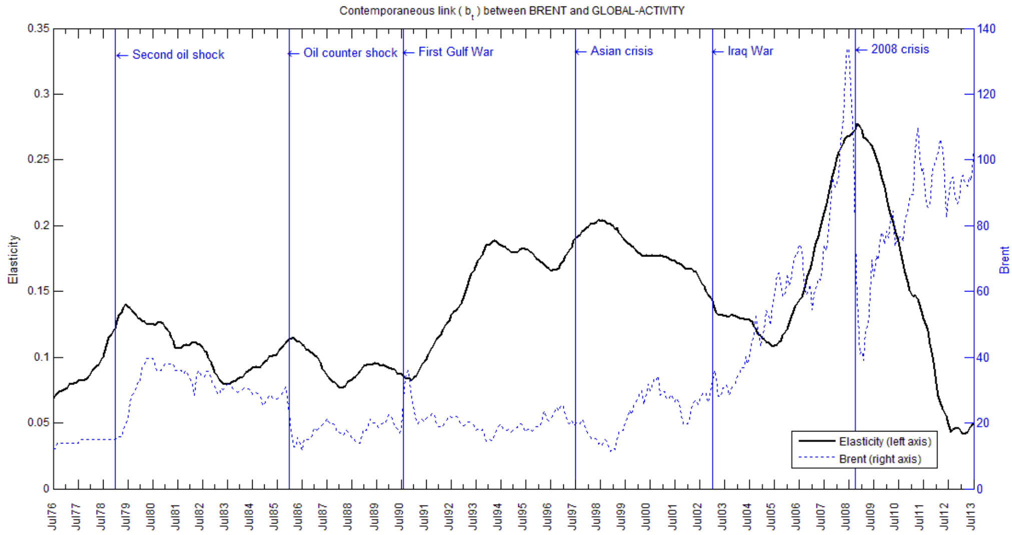



*Source: Authors calculation*

### 4.1.3 Elasticity between Global Activity and Brent

As shown in figure 3, the relationship between the global activity and the crude oil price is positive during all our sample period and reinforced between 1991 and 1999. This illustrate the prominent role of demand as a driver of oil price as argued by Kilian (2009) and Kilian et al. (2009). The steady increase in the elasticity until the 2008 world trade collapse (from 0.07 to 0.27) illustrate the short run vertical oil supply curve and inelastic oil demand stated in the literature. Namely, Baumeister and Peersman (2013) argue that these findings are mainly explained by the efficiency gains observed since the second oil shock and by changes in the total composition of oil demand. The increasing outsourcing of industrialized production towards emerging countries where oil prices are subsidized by governments leads to an oil demand less responsive to oil prices. This is reinforced by the fact that the industrialized countries GDPs are now oriented to tertiary sector and to efficiency gains, namely for transportion sector.

Figure 3: Global Activity elasticity of Brent



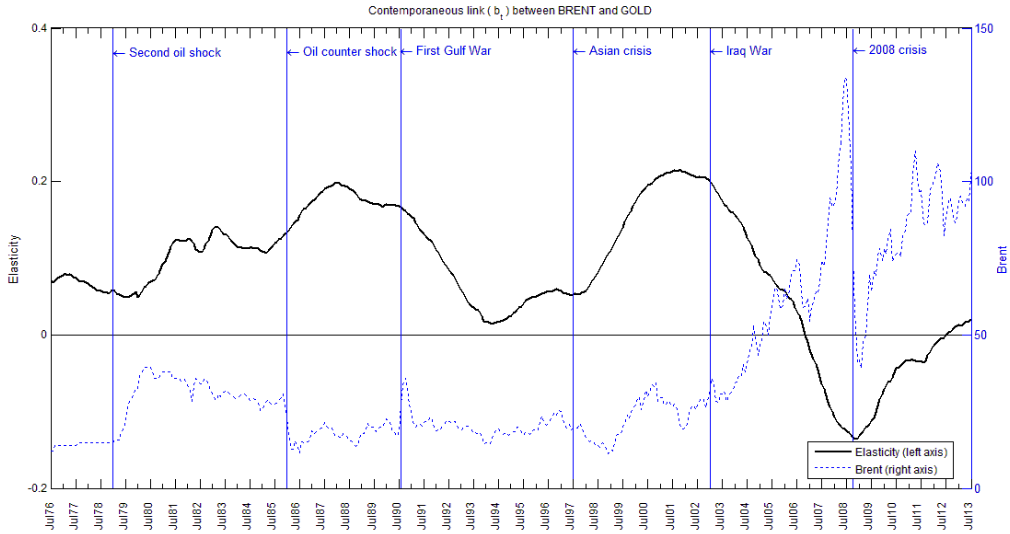

*Source: Authors calculation*

### 4.1.4 Elasticity between Gold and Brent

As shown in figure 4, the relationship between gold price and the crude oil price is quite instable. The elasticity which was positive from 1976 to 2007 became negative afterward. The shape of the gold elasticity of brent follows the shape of the effective exchange rate of the dollar with a sharp appreciation between 1979 and 1985 and between 1994 and 2002, and a depreciation from 1985 to 1994 and from 2003 to 2008. It then highlights the relationship between these 3 variables : the crude oil price, the gold price and the effective exchange rate of the dollar.

The 2002-2013 period is very interesting. In the wake of the financial crisis of 2007-2008, the crude oil price collapsed from 145 USD to less than 40 USD at the end of 2008. During the same time the Gold price registered a sharp increase. Two elements can explain this fact. On the one hand, we can consider that the traditionnal safe haven asset theory reinforced during this post-crisis period and helped to boost the gold prices in the market. On the other hand, it could be explained by the growing monetization of the gold market.

Figure 4: Gold Elasticity of Brent




*Source: Authors calculation*

## 4.2 Forecast Error Variance Decomposition (FEVD)

The Forecast Error Variance Decomposition (henceforth FEVD) in figure 5 measures the contribution of different variables in the model to the volatility of the variable of interest. They are calculated for the whole sample of estimation. It should contribute to a more comprehensive analysis of our subject with regards to time-changing economic environment pattern and monetary policy during the estimation period.

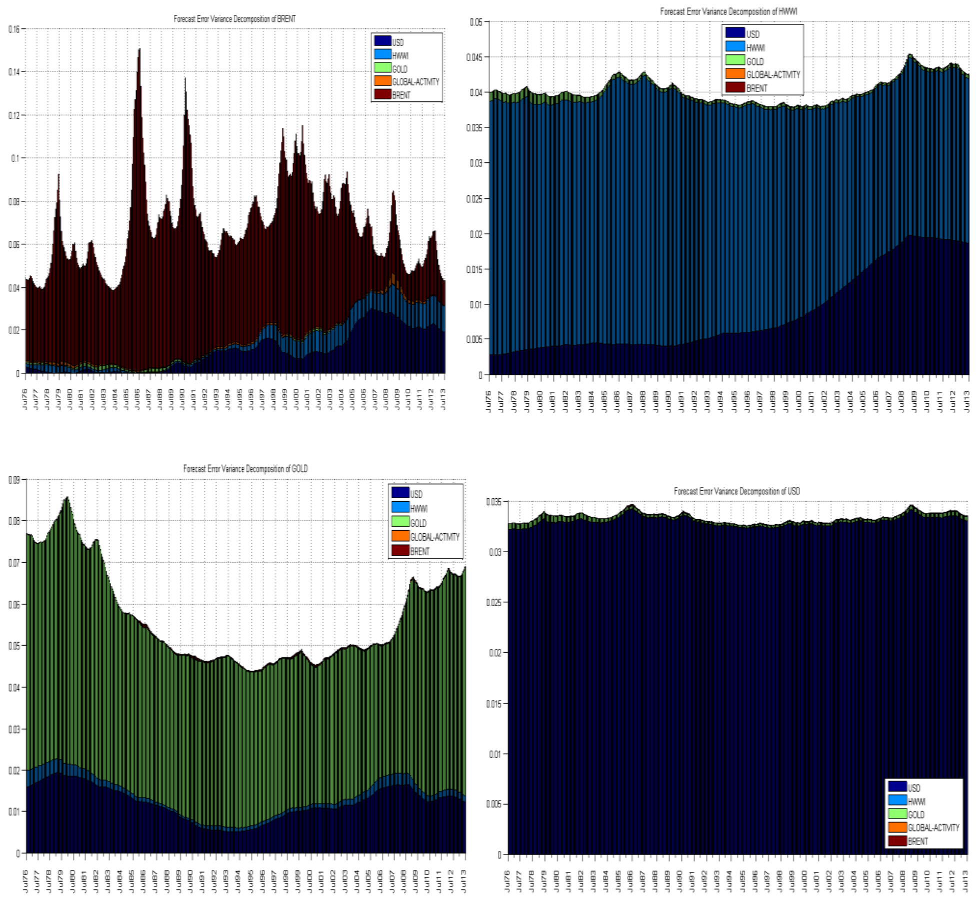
Regarding the FEVD of Brent, several conclusions emerge. First, oil price volatility is time-varying. Mainly, there is a sustained increase of oil price volatility from 1976 to 2001 as evidenced in the literature with a peak on 1986 and 1990. Thereafter, the volatility of the Brent decreases despite the rise in the economic development of the BRICs economies and more especially China. Second, with the exception of some abnormal price movements occurring in the market (1979, 1986, 1991 and 2008), Brent variance is less and less self-explained. Third, USD and HWWI variance impacts are more and more important, especially after the beginning of the 21st century.

The gold FEVD is globally explained by its self-variance (and so fundamentals) and also by USD variance, as this commodity is priced in USD.

The HWWI is mainly explained by two variances. Thus, while the share of USD variance explanation ramped up since the end of 1990s (reaching half of the explanation), the explanation through the HWWI variance decreases from around 90% to 50%. This result is interesting, considering the fact that the majority of commodities involved in HWWI composition are priced in USD. Regarding the

FEVD of USD, we can note that USD variance is remarkably self-explained and flat since 1976. This might be explained by the fact that the direction of bilateral USD exchange rate changes after an oil price changes depends on the considered country. Lizardo and Mollick (2010) argue that currencies of oil exporting countries appreciate relative to the US dollar while those of oil importing countries depreciate relative to the US Dollar following a rise in oil price. Thus, the final effect of oil price changes on the effective exchange rate of US Dollar considered in this study might be dampened.

Figure 5: Forecast Error Variance Decomposition (FEDV)




*Source: Authors calculation*

## 5. Conclusion

In the wake of the recent literature that assesses the relationship between oil market and the economy, we take a further step in this study by studying the link that exists between oil prices, financial variables and commodity prices. More particularly, we analyse the oil-dollar link in order to understand the historical coincidence between crude oil and financial shock and crises. As the recent empirical literature advances, we find an important contribution of global economic activity in explaining the dynamics of commodity prices, in particular oil prices. However, the contribution of this paper to the literature is three-folds. First, we demonstrate that both the elasticity of crude oil prices with respect to a set of variables and the forecast error variance decomposition (FEVD) of these variables vary across time. This result gives a formal estimation of the unstable relationship between oil and the economy, namely the link between oil prices and the USD. Second, we show that a depreciation of the effective exchange rate of the dollar leads to an oil price increase, except during 1979-1989 period. It is in accordance with the recent view that oil price is endogenous with respect to global economic activity. Indeed, as the oil price is labeled in USD, depreciation of the dollar stimulated oil demand, namely from emerging market. Third, as the time-varying nature of the estimated elasticities permits to uncover, we also highlight the increasing role of the other commodities markets in the oil price dynamics. The HWWI index volatility can be considered as an important factor to understand the oil prices dynamics. These results are confirmed by the FEVD analysis, reflecting that the HWWI and the effective exchange rate of the dollar variances contribute to the explanation of the Brent price volatility.

Taking all these results into account leads us to conclude that the previous studies only based on the US economy and the oil price should be deepened. The world economic environment has also undergone profound upheavals in the past 10 years, with the emerging countries establishing themselves firmly as drivers of world economic growth. The increase in global oil consumption is thus being generated mainly by these countries since the beginning of 2000. "Currency war" that occurs since the financial crisis and the changes in oil price setting triggered by the non-conventional oil production in the US open new potential horizon to understand the oil-dollar link. Moreover, monetary policies from ECB, FED or Bank of Japan (Quantitative Easing policies) may impact the global comprehension of the financial markets and also the evolution of emerging countries (such as China, Turkey, Indonesia, Brazil) which are more and more the new players of commodity market.

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## Appendices:

### A. HWWI Commodity Price Index Methodology

The HWWI index is a monthly commodity price index developed by the Hamburg Institute of International Economics (HWWI) in Germany. The index measures the changes observed in the commodity prices and is considered as an indicator of developments in the cost of imported raw materials. The HWWI index is split in different sub-indexes: HWWI index TOTAL, HWWI index excluding energy, HWWI energy raw materials, HWWI industrial raw materials, HWWI Food Total. According to AIECE Report (2013), the weights for individual commodities are based on their share in total commodities imports of the OECD countries, excluding intra-EU trade.

The weights of the different commodities and commodity groups in the HWWI index we used in our study are the following:

| per cent share in: | total | excl. energy | | total | excl. energy |
|---|---|---|---|---|---|
| **HWWI index, total** | 100 | | **Industrial raw materials** | 15,4 | 73,8 |
| **Total excl. energy** | 20,8 | 100 | Agricultural raw materials | 4,3 | 20,6 |
| | | | - Cotton | 0,1 | 0,6 |
| **Food total** | 5,5 | 26,2 | - Wool | 0,1 | 0,4 |
| | | | - Hides | 0,1 | 0,7 |
| Cereals | 1,4 | 6,9 | - Natural rubber | 0,8 | 3,9 |
| - Barley | 0,0 | 0,2 | - Wood | 1,8 | 8,9 |
| - Maize | 0,7 | 3,4 | - Woodpulp | 1,3 | 6,1 |
| - Wheat | 0,5 | 2,3 | | | |
| - Rice | 0,2 | 0,9 | Non-ferrous metals | 7,9 | 37,9 |
| | | | - Aluminium | 3,7 | 17,6 |
| Oilseeds, vegetable oils | 1,9 | 9,1 | - Copper | 2,5 | 12,2 |
| - Soybeans | 0,7 | 3,5 | - Lead | 0,2 | 0,8 |
| - Soybean meal | 0,8 | 3,7 | - Nickel | 0,9 | 4,4 |
| - Soybean oil | 0,1 | 0,2 | - Tin | 0,2 | 0,9 |
| - Coconut oil | 0,1 | 0,4 | - Zinc | 0,4 | 2,0 |
| - Palm oil | 0,2 | 0,8 | | | |
| - Sunflower oil | 0,1 | 0,5 | Iron ore, steel scrap | 3,2 | 15,3 |
| | | | - Iron ore | 2,2 | 10,8 |
| Tropical beverages, sugar | 2,1 | 10,3 | - Steel scrap | 0,9 | 4,5 |
| - Coffee | 1,2 | 5,6 | | | |
| - Cocoa | 0,5 | 2,2 | **Energy raw materials** | 79,2 | |
| - Tea | 0,2 | 0,7 | - Coal | 4,5 | |
| - Sugar | 0,4 | 1,8 | - Crude oil | 74,6 | |

[1] Based on world imports of OECD countries minus Intra-EU trade, 2005-2007

**B. Unit Root Tests**

Before implementing the estimation, we proceed to three different unit root tests to determine the order of integration of each series: Augmented Dickey-Fuller (1981) (ADF), Philips and Perron (1988) (PP) and the Zivot and Andrews (1992) tests. Relying on the ZA test allow us to account for the presence of structural breaks and then consolidate our results. For the ADF and PP tests, the null hypothesis is the non stationarity. For the Zivot and Andrews procedure, the null hypothesis is the presence of a unit root without any exogenous structural break, while the alternative hypothesis is characterized by the stationarity with a break date endogenously determined. The structural breaks for the two elements are tested on the trend and on the constant term. As shown in Table 1, our series are integrated of order 1. We thus first-differentiate our variables in order to control for non-stationarity in our estimation.

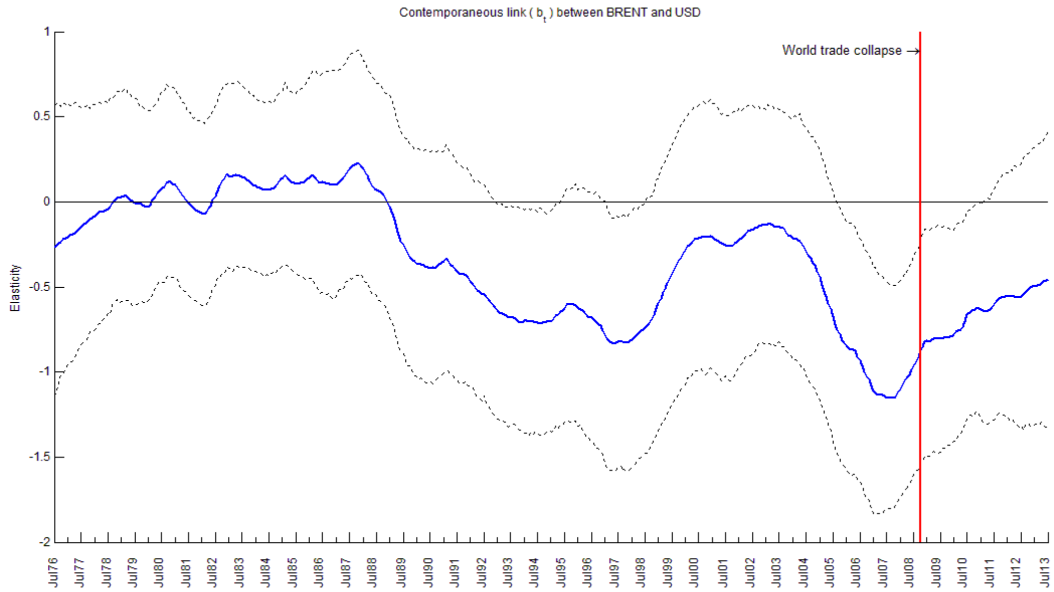
**Table 1. Results of ADF, Philips-Perron & Zivot and Andrews tests**

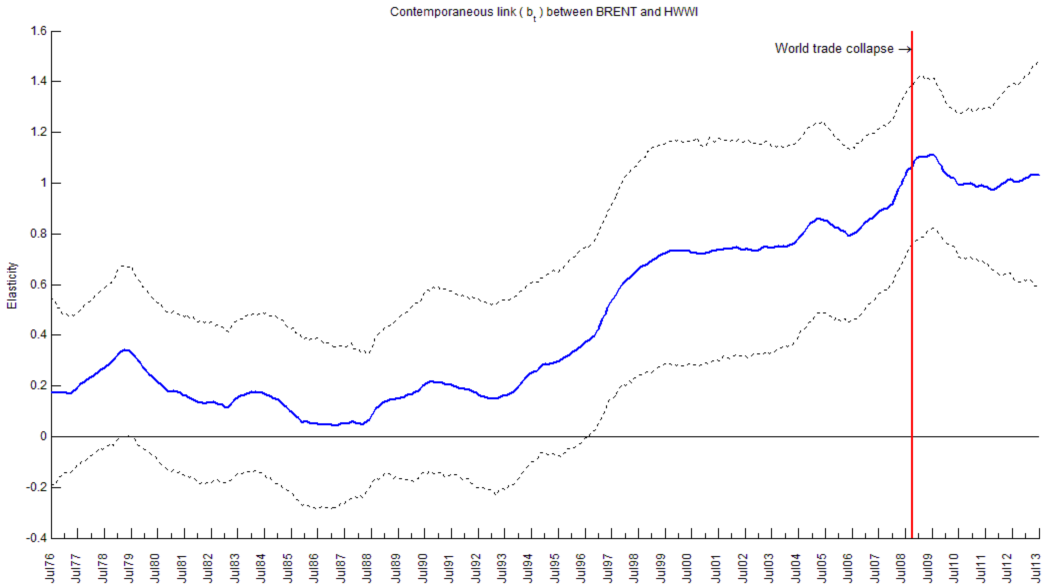
| 1976m7-2013m7 | **Brent** | **USD** | **Gold** | **HWWI** |
|---|---|---|---|---|
| **ADF** | I(1) | I(1) | I(1) | I(1) |
| **PP** | I(1) | I(1) | I(1) | I(1) |
| **Z&A** | I(1) | I(1) | I(1) | I(1) |

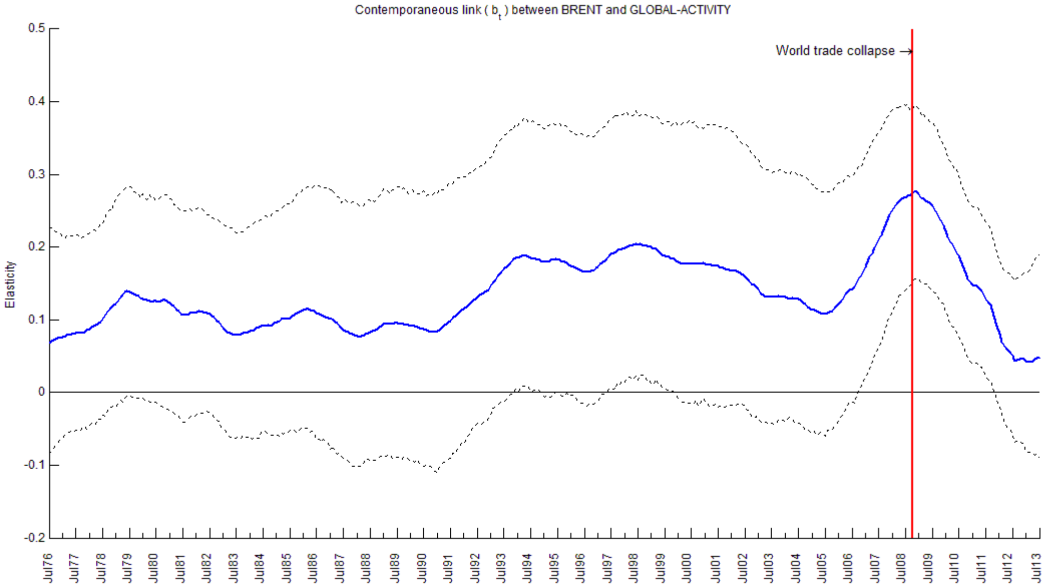
I(0) (resp. I(1)): series are integrated of order 0 (resp. 1).

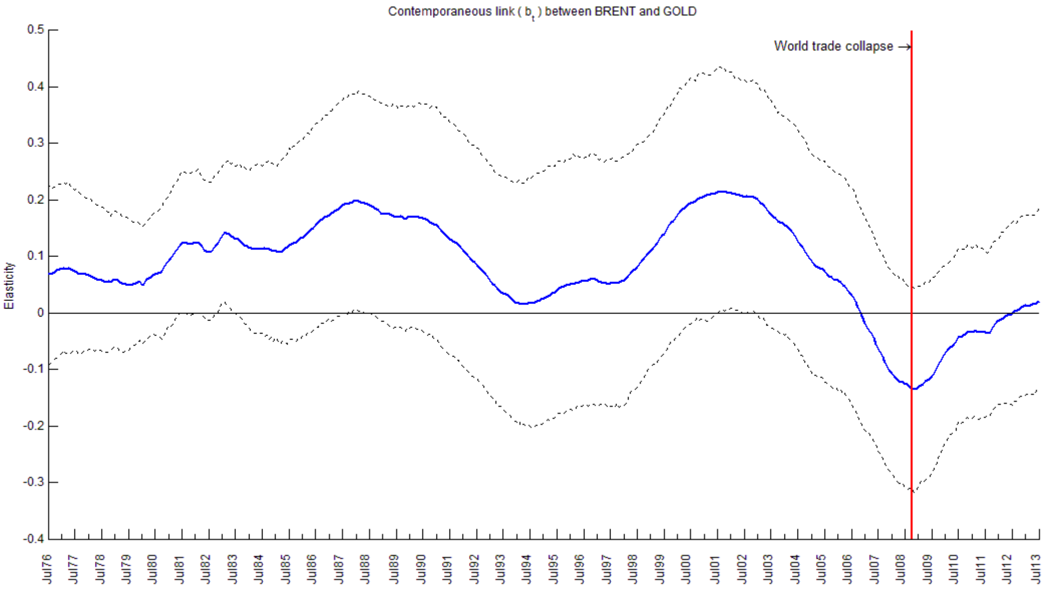
## C. Figures

Figure 6: Elasticities with confidence intervals




Contemporaneous link ( b_t ) between BRENT and GLOBAL-ACTIVITY
World trade collapse →
Elasticity


Contemporaneous link ( b_t ) between BRENT and GOLD
World trade collapse →
Elasticity

## D. Time-varying VAR model

### D.1 State-space representation

In order to estimate the parameters of interest, the reduced form VAR model (2) can be rewritten as:

$$\begin{aligned} Y_t &= \begin{pmatrix} c_t & A_{1,t} & \cdots & A_{p,t} \end{pmatrix} \begin{pmatrix} 1 & Y'_{t-1} & \cdots & Y'_{t-p} \end{pmatrix}' + \epsilon_t \\ &= A_t Z_{t-1} + \epsilon_t \end{aligned}$$

Vectorization of the last equation yields

$$Y_t = \left( Z'_{t-1} \otimes I_n \right) \alpha_t + \epsilon_t \tag{5}$$

where $\alpha_t$ is a column vector that contains stacked columns of matrix $A_t$ and $\otimes$ denotes the Kronecker products.

### D.1.1 Reduced form VAR

State space representation of the reduced form VAR representation (2) can be obtained using (4) and the time paths of lag coefficients $\alpha_t$ (5). That is,

$$\begin{cases} \alpha_t = \alpha_{t-1} + \omega_t \\ Y_t = \left( Z'_{t-1} \otimes I_n \right) \alpha_t + B_t^{-1} \Sigma_t v_t \end{cases} \tag{6}$$

where the residuals of state and measurement equations are assumed to be normally distributed. That is,

$$\omega_t \rightsquigarrow \mathcal{N}(0, Q) \;\; v_t \rightsquigarrow \mathcal{N}(0, \mathbf{I}_n)$$

Knowing that innovations $\{w_t, v_t\}_{t=1}^{T}$ are multivariate Gaussian, if the initial value of VAR parameters $\alpha_0$ is also Gaussian, the entire sequences of $\{\alpha_t\}_{t=1}^{T}$ can be generated *via* forward and backward recursion of Kalman filter conditional on[10]

[10] Or, in other words, assuming that the entire sequence of contemporaneous coefficients and standard errors $\{B_t, \Sigma_t\}_{t=1}^{T}$ are known or given, as well as the variance-covariance matrix Q.

$Y_t, B_t, \Sigma_t$ and Q. Therefore, as shown in Cogley and Sargent (2005), joint posterior density for VAR parameters is a product of independent normal distribution. That is,

$$p\left(\alpha_T|Y_T, B_T, \Sigma_T, Q\right) = f\left(\alpha_T|Y_T, B_T, \Sigma_T, Q\right) \prod_{t=1}^{T-1} f\left(\alpha_t|\alpha_{t+1}, Y_t, B_t, \Sigma_t, Q\right) \tag{7}$$

where both posterior density $p(\cdot)$ and density functions $f(\cdot)$ are linear Gaussian with means and variances given by

$$\begin{aligned} \alpha_{t|t+1} &= E\left(\alpha_t|\alpha_{t+1}, Y_t, B_t, \Sigma_t, Q\right) \\ P_{t|t+1} &= Var\left(\alpha_t|\alpha_{t+1}, Y_t, B_t, \Sigma_t, Q\right) \end{aligned}$$

### D.1.2 Contemporaneous coefficients

Let us define

$$\hat{Y}_t = Y_t - \left(Z'_{t-1} \otimes I_n\right) \alpha_t = B_t^{-1} \Sigma_t v_t$$

where $\hat{Y}_T$ will be observable when $\alpha_t$ is known or given. Since $B_t$ is lower triangular matrix with ones along the diagonal and $\Sigma_t$ is a diagonal matrix, the equality $B_t \hat{Y}_t = \Sigma_t v_t$ can be written by equation as

$$\begin{aligned} \hat{Y}_{2t} &= -\hat{Y}_{1t} b_{21,t} + \sigma_{2,t} v_{2t} \\ \hat{Y}_{3t} &= -\left(\hat{Y}_{1t}\ \hat{Y}_{2t}\right) \begin{pmatrix} b_{31,t} \\ b_{32,t} \end{pmatrix} + \sigma_{3,t} v_{3t} \\ \vdots &= \vdots \\ \hat{Y}_{nt} &= -\left(\hat{Y}_{1t}\ \hat{Y}_{2t}\ \cdots\ \hat{Y}_{n-1t}\right) \begin{pmatrix} b_{n1,t} \\ b_{n2,t} \\ \vdots \\ b_{nn-1,t} \end{pmatrix} + \sigma_{n,t} v_{nt} \end{aligned}$$

Moreover, under the block diagonality assumption of $S$ and conditional on $Y_t$, $\alpha_t$, $\Sigma_t$, $S$ and $I_n$, each block of the vector of contemporaneous coefficients $b_t$ is computed *via* forward and backward recursion of Kalman filter applied to state space representation of the corresponding equation. That is, for $i = 2, \dots, n$

$$\begin{cases} b_{[i]t} = b_{[i]t-1} + \zeta_{[i]t} \\ \hat{Y}_{it} = -\hat{Y}_{[1,\dots,i-1]t} b_{[i]t} + \sigma_{i,t} v_{it} \end{cases} \tag{8}$$

where $b_{[i]t}$ is the block of the vector $b_t$ corresponding to the $i^{th}$ equation, $\zeta_{[i]t}$ is the $i^{th}$ block of the vector of state errors corresponding to $b_{[i]t}$ with

$$\zeta_{[i]t} \rightsquigarrow \mathcal{N}(\mathbf{0}, S_{[i]})$$

where $S_{[i]}$ is the $i^{th}$ block of the matrix $S$, and $\hat{Y}_{[1,\dots,i-1]t}$ denotes a row vector $\hat{Y}_{1t}\ \hat{Y}_{2t} \cdots \hat{Y}_{i-1t}$. Like VAR parameters in the previous subsection, joint posterior density for contemporaneous coefficients is given by

$$\begin{aligned} p\left(b_{[i]T} | \hat{Y}_{[1,\dots,i]T}, \sigma_{i,T}, S_{[i]}\right) &= f\left(b_{[i]T} | \hat{Y}_{[1,\dots,i]T}, \sigma_{i,T}, S_{[i]}\right) \\ &\quad \prod_{t=1}^{T-1} f\left(b_{[i]t} | b_{[i]t+1}, \hat{Y}_{[1,\dots,i]T}, \sigma_{i,T}, S_{[i]}\right) \end{aligned} \tag{9}$$

where posterior density $p(\cdot)$ and density functions $f(\cdot)$ are linear Gaussian with means and variances given by

$$\begin{aligned} b_{[i]t|t+1} &= E\left(b_{[i]t} | b_{[i]t+1}, \hat{Y}_{[1,\dots,i]T}, \sigma_{i,T}, S_{[i]}\right) \\ \Lambda_{[i]t|t+1} &= Var\left(b_{[i]t} | b_{[i]t+1}, \hat{Y}_{[1,\dots,i]T}, \sigma_{i,T}, S_{[i]}\right) \end{aligned}$$

### D.1.3 Standard error coefficients

Let us define

$$Y_t^* = B_t \hat{Y}_t = \Sigma_t \upsilon_t \tag{10}$$

It is worth noting that when $B_t$ is known (or conditional on $B_t$), $Y_t^*$ is observable. Unlike measurement equation above, this is a non-linear system. However, it can be easily transformed into a linear one by squaring and taking natural logarithm of each equation in (10). That is, for the $i^{th}$ equation,

$$\begin{aligned} Y_{it}^* &= \sigma_{i,t}\upsilon_{it} \\ \left(Y_{it}^*\right)^2 &= \left(\sigma_{i,t}\upsilon_{it}\right)^2 \\ \ln\left(\left(Y_{it}^*\right)^2\right) &= 2\ln\left(\sigma_{i,t}\right) + \ln\left(\upsilon_{it}^2\right) \end{aligned}$$

Notice that the element $(Y_{it}^*)^2$ might be very small. Thus, in order to have more robust estimation, it is necessary to add $(Y_{it}^*)^2$ with a constant correction $\bar{c} = 0.001$. That is,

$$\begin{aligned} \ln\left(\left(Y_{it}^*\right)^2 + \bar{c}\right) &= 2\ln\left(\sigma_{i,t}\right) + \ln\left(\upsilon_{it}^2\right) \\ Y_{it}^{**} &= 2h_{it} + u_{it} \end{aligned}$$

State space representation of the vector of standard-error coefficients $h_t$ is therefore given by

$$\begin{cases} h_t = h_{t-1} + \eta_t \\ Y_t^{**} = 2h_t + u_t \end{cases} \tag{11}$$

where

$$\eta_t \rightsquigarrow \mathcal{N}(\mathbf{0}, W).$$

It is worth noting that measurement errors $u_t$ no longer follow a normal distribution. Instead, they are distributed as $\ln(\chi^2(1))$. However, as in Kim et al. (1998), $\ln(\chi^2(1))$ distribution can be approximated, according to a $(n \times T)$ matrix of indicator variables $D_t = (d_1\, d_2 \cdots d_T)$, by a mixture of 7 normal distributions with probabilities $q_j = Pr(w = j)$ for $j = 1, \ldots, 7$, means $E(w) = m_j - 1.2704$, and variances $Var(w) = \nu_j^2$. The parameters of the relevant distribution are selected from the following table

Mixture Distribution

| $\omega$ | $q_j$ | $m_j$ | $\nu_j^2$ |
|---|---|---|---|
| 1 | 0.00730 | -10.12999 | 5.7956 |
| 2 | 0.10556 | -3.97281 | 2.61369 |
| 3 | 0.00002 | -8.56686 | 5.17950 |
| 4 | 0.04395 | 2.77786 | 0.16735 |
| 5 | 0.34001 | 0.61942 | 0.64009 |
| 6 | 0.24566 | 1.79518 | 0.34023 |
| 7 | 0.25750 | -1.08819 | 1.26261 |

Source: Kim et *al.* (1998)

Conditional on $Y_t^{**}$ and $W$, joint posterior density of standard error vector of parameters is given by

$$p\left(h_T | Y_T^{**}, W\right) = f\left(h_T | Y_T^{**}, W\right) \prod_{t=1}^{T-1} f\left(h_t | h_{t+1}, Y_t^{**}, W\right) \tag{12}$$

where posterior density $p(\cdot)$ and density functions $f(\cdot)$ are now linear Gaussian with means and variances given by

$$\begin{aligned} h_{t|t+1} &= E\left(h_t | h_{t+1}, Y_t^{**}, W\right) \\ H_{t|t+1} &= Var\left(h_t | h_{t+1}, Y_t^{**}, W\right) \end{aligned}$$

It is worth noting that given $Y_t^{**}$ and $h_t$, a new selection matrix $D_t$ can be generated by sampling from

$$P\left(d_{it} = k | Y_{it}^{**}, h_{it}\right) \propto q_j f_N\left(Y_{it}^{**} | 2h_{it} + m_j - 1.2704, \nu_j^2\right)$$

for $j = 1, \ldots, 7$ and $i = 1, \ldots, n$.

### D.2 Kalman filter and Gibbs sampling

Consider the linear Gaussian state space model

$$\begin{cases} \xi_t = \mathbf{T}\xi_{t-1} + \mathbf{c} + \mathbf{R}\eta_t \\ Y_t = \mathbf{Z}_t\xi_t + \mathbf{d} + \varepsilon_t \end{cases}$$

where $\xi_t$ is the unobservable state vector, $Y_t$ is the vector of observations, and $\eta_t$ and $\varepsilon_t$ are the vectors of serially uncorrelated disturbances with mean 0 and covariance matrices Q and $H_t$. That is,

$$\begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \rightsquigarrow \mathcal{N}\left( \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_t \end{bmatrix} \right)$$

#### D.2.1 Forward recursion of Kalman filter

Define

$$\begin{aligned} \xi_{t|t-1} &= E\left\{\xi_t | Y_{t-1}, \mathbf{Z}_{t-1}, \mathbf{H}_{t-1}, \mathbf{Q}\right\} \\ P_{t|t-1} &= E\left\{ \left(\xi_t - \xi_{t|t-1}\right) \left(\xi_t - \xi_{t|t-1}\right)' \right\} \end{aligned}$$

where $\xi_{t|t-1}$ denotes the linear projection of $\xi_t$ on $Y_{t-1}$, $Z_{t-1}$, $H_{t-1}$, Q and a constant (the forecast of $\xi_t$), and $P_{t|t-1}$ the mean squared error associated with the forecast. We assume that the initial state is Gaussian, that is

$$\xi_0 \rightsquigarrow \mathcal{N}\left(\xi_{0|0}, P_{0|0}\right)$$

Therefore, given the initial values $\xi_{0|0}$ and $P_{0|0}$ , Kalman filter recursion is given by

$$
\begin{aligned}
\xi_{t|t-1} &= \mathbf{T}\xi_{t-1|t-1} + \mathbf{c} \\
P_{t|t-1} &= \mathbf{T}P_{t-1|t-1}\mathbf{T}' + \mathbf{RQR}' \\
Y_{t|t-1} &= \mathbf{Z}_t\xi_{t|t-1} + \mathbf{d} \\
F_t &= \mathbf{Z}_t P_{t|t-1}\mathbf{Z}_t' + \mathbf{H}_t \\
K_t &= P_{t|t-1}\mathbf{Z}_t' F_t^{-1} \\
\xi_{t|t} &= \xi_{t|t-1} + K_t\left(Y_t - Y_{t|t-1}\right) \\
P_{t|t} &= \left(\mathbf{I} - K_t\mathbf{Z}_t\right)P_{t|t-1}\left(\mathbf{I} - K_t\mathbf{Z}_t\right)' + K_t\mathbf{H}_t K_t'
\end{aligned}
$$

For $t = 1, \dots, T$.

### D.2.2 Backward recursion using Gibbs sampling

Forward recursion of Kalman filter in the previous section yields the following sequences of updated and forecasted state variables and variances $\{\xi_{t|t}\}_{t=1}^{T}$, $\{\xi_{t|t-1}\}_{t=1}^{T}$, $\{P_{t|t}\}_{t=1}^{T}$, and $\{P_{t|t-1}\}_{t=1}^{T}$. Define the smoothed estimates, that is the estimates based on all available observations, of states and variances as

$$
\begin{aligned}
\xi_{t|T} &= E\left\{\xi_t | Y_T, \mathbf{Z}_T, \mathbf{H}_T, \mathbf{Q}\right\} \\
P_{t|T} &= E\left\{\left(\xi_t - \xi_{t|T}\right)\left(\xi_t - \xi_{t|T}\right)'\right\} \\
\xi_t &\rightsquigarrow \mathcal{N}\left(\xi_{t|T}, P_{t|T}\right)
\end{aligned}
$$

Therefore, last entry of updated sequences of state variables and variances, $\xi_{T|T}$ and $P_{T|T}$, are respectively the mean and variance of the smoothed estimate for the final date in the sample. That is, smoothed estimate for $T$ is drawn from

$$
\xi_T \rightsquigarrow \mathcal{N}\left(\xi_{T|T}, P_{T|T}\right)
$$

Following Carter and Kohn (1994), entire sequences of the smoothed estimates are obtained by moving backward through the sample starting from $t = T - 1$ to $t = 1$ using the following set of equations:

$$
\begin{aligned}
L_t &= P_{t|t}\mathbf{T}'\left(\mathbf{T}P_{t|t}\mathbf{T}' + \mathbf{RQR}'\right)^{-1} \\
\xi_{t|t+1} &= \xi_{t|t} + L_t\left(\xi_{t+1} - \mathbf{c} - \mathbf{T}\xi_{t|t}\right) \\
P_{t|t+1} &= \left(\mathbf{I} - L_t\mathbf{T}\right)P_{t|t}\left(\mathbf{I} - L_t\mathbf{T}\right)' + L_t\mathbf{RQR}'L_t' \\
\xi_t &\rightsquigarrow \mathcal{N}\left(\xi_{t|t+1}, P_{t|t+1}\right)
\end{aligned}
$$

## D.3 Bayesian inference

The entire sequence of parameters of interest $A_t$, $B_t$ and $\Sigma_t$ are estimated by simulating their distribution using Bayesian approach, namely we implement a Gibbs sampler. Simulation is carried out in four steps, simulating in turn time-varying reduced form VAR parameters $A_t$, contemporaneous coefficients $B_t$, volatilities $\Sigma_t$ and variance-covariance matrix V.

### D.3.1 Priors

Specifications of prior distribution in this paper follow Primiceri (2005). Initial value for time-varying parameters and variance-covariance matrices are assumed to be mutually independent. An initial subsample of 40 observations is used to generate OLS point estimates of the parameters of interest. Priors of the initial value of the reduced form VAR parameters $A_0$, the contemporaneous coefficients $B_0$ and the logarithm of volatilities $\ln(\Sigma_0)$ are assumed to follow a normal distribution with mean equals to the corresponding OLS estimates of the parameter and variance equals to four times the corresponding OLS variance for $A_0$ and $B_0$, and equals to the identity matrix for $\ln(\Sigma_0)$.That is,

$$
\begin{aligned}
\alpha_0 &\rightsquigarrow \mathcal{N}\left(\hat{\alpha}_{ols}, 4\cdot V\left(\hat{\alpha}_{ols}\right)\right) \\
b_0 &\rightsquigarrow \mathcal{N}\left(\hat{b}_{ols}, 4\cdot V\left(\hat{b}_{ols}\right)\right) \\
h_0 &\rightsquigarrow \mathcal{N}\left(\hat{h}_{ols}, \mathbf{I}_n\right)
\end{aligned}
$$

Priors of different blocks of the variance-covariance matrix V, in turn, are assumed to be independent and to follow an inverted Wishart distribution. That is,

$$
\begin{aligned}
Q &\rightsquigarrow \mathcal{IW}\left(k_Q^2 \cdot 40 \cdot V\left(\hat{b}_{ols}\right), 40\right) \\
S_{[i]} &\rightsquigarrow \mathcal{IW}\left(k_S^2 \cdot (i+1) \cdot V\left(\hat{b}_{ols}\right), (i+1)\right) \\
W &\rightsquigarrow \mathcal{IW}\left(k_W^2 \cdot (n+1) \cdot \mathbf{I}_n, (n+1)\right)
\end{aligned}
$$

where $k_Q^2 = 0.01$, $k_S^2 = 0.1$, $k_W^2 = 0.01$ and n is the number of endogenous variables in the system. Notice that these priors assumptions together with random walk assumption in (5) imply normal priors on the entire sequences of of $A_t$, $B_t$ and $\Sigma_t$ conditional on Q, S and W. Set in this way, as it is explained in Primiceri (2005), priors are not flat, but sufficiently diffuse and uninformative, letting data determine the best estimates of given parameters.

### D.3.2 Posterior distribution

Given that the state space models (6), (8) and (11) are linear and Gaussian, posterior distributions of the state variables $\alpha_t|Y_t, B_t, \Sigma_t, S$, $b_t|Y_t, \alpha_t, \Sigma_t, S$ and $h_t|Y_t, \alpha_t, B_t, W$ are generated using forward and backward recursion of Kalman filter developed in 1.1.3. Variance-covariance matrices Q, S and W are generated from their respective independent posterior distributions, which are, assume to follow an inverted Wishart distribution. That is,

$$
\begin{aligned}
Q|Y_t, A_t, B_t, \Sigma_t &\rightsquigarrow \mathcal{IW}\left(\left(\sum_{t=p+1}^{T} \omega_t \omega_t' + \underline{Q}\right), (T - p + \underline{q})\right) \\
S_{[i]}|Y_t, A_t, B_t, \Sigma_t &\rightsquigarrow \mathcal{IW}\left(\left(\sum_{t=p+1}^{T} \zeta_{[i]t} \zeta_{[i]t}' + \underline{S_{[i]}}\right), \left(T - p + \underline{s_{[i]}}\right)\right) \\
W|Y_t, A_t, B_t, \Sigma_t &\rightsquigarrow \mathcal{IW}\left(\left(\sum_{t=p+1}^{T} \eta_t \eta_t' + \underline{W}\right), (T - p + \underline{w})\right)
\end{aligned}
$$

where $\underline{Q}$, $\underline{S_{[i]}}$ and *W* are positive definite scale matrices from the inverted Wishart prior distributions of Q, block matrix $S_{[i]}$ of *S* and *W*, and *q*, $s_{[i]}$, *w* their respective degree of freedom.

### D.3.3 Markov Chain Monte Carlo (MCMC) algorithm

To resume, the Markov Chain Monte Carlo (MCMC) algorithm takes the following form:

1. Specify the initial sequence of $A_t$, $B_t$, $\Sigma_t$ , $D_t$ and V

2. Generate the states $\alpha_t$ conditional on $Y_t, B_t, \Sigma_t$ and Q using Kalman filter for $t = 1, \dots, T$
3. Generate off-diagonal elements $b_t$ of the contemporaneous matrix $B_t$ conditional on $Y_t,\ \alpha_t, \Sigma_t$ and *S* using Kalman filter for $t = 1, \dots, T$
4. Generate volatilities $\sigma_t$ conditional on $Y_t,\ \alpha_t,\ b_t, D_t$ and W using Kalman filter for $t = 1, \dots, T$
5. Generate a new selection matrix $D_t$ by sampling from $P(d_{it} = k|Y_{it}^{**}, h_{it})$ conditional on $Y_t, \alpha_t, b_t, \sigma_t$ for $t = 1, \dots, T$
6. Generate variance-covariance matrix V by sampling from independent inverted Wishart distribution
7. Check for stationnarity of the VAR, and if, only if it is the case, store parameters of interest
8. Go to step 2

It is worth noting that step 7 is implemented in order to insure that realizations of the VAR are stationary and only draws are accepted and stored. Stationnarity is checked by calculating the maximum absolute eigenvalue of the companion matrix corresponding to a VAR(1) representation of VAR(p) at each point in time. If the maximum absolute eigenvalue is strictly lower than unity, VAR are stationary and draws are accepted.

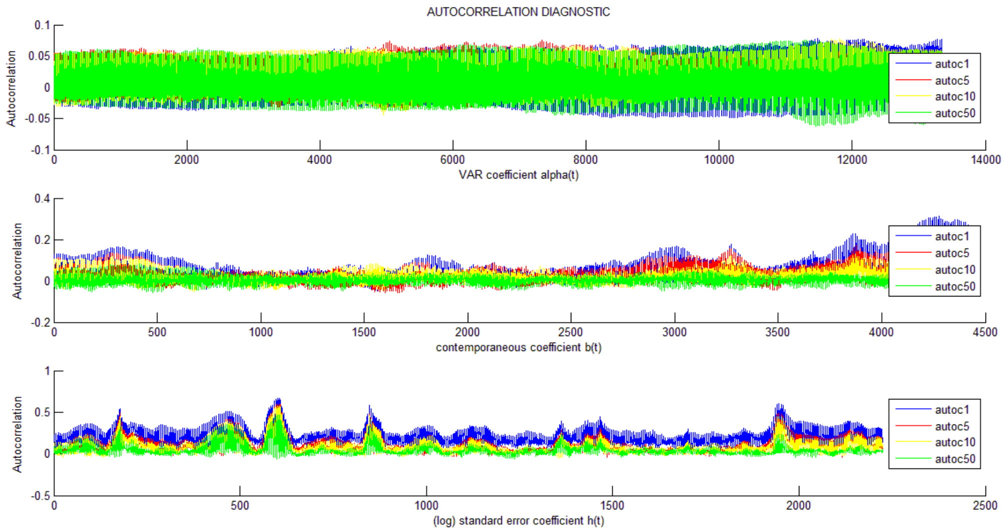
## E. Convergence Diagnostic

In order to assess convergence of the sample to the posterior distribution, we perform a set of standard convergence diagnostic tests namely the autocorrelation function and the Raferty and Lewis (1995) MCMC diagnostics (numbers of draws and I-statistic).[11] All figures are subdivided in three parts and depict respectively the convergence diagnostic tests for the lag-coefficients ($\alpha_t$) of the model (2), the contemporaneous coefficients ($b_t$) and the (log) standard error coefficients ($h_t$). The unit of x-axis correponds to the number of estimated coefficients for all time periods. For instance, the number 2225 (=5x445) corresponds to the estimated values of the (log) standard error coefficients $h_t = \left(h_t^{usd}, h_t^{hwwi}, h_t^{gold}, h_t^{drycargo}, h_t^{brent}\right)$ for the period 1976M7 to 2013M7. In sum, convergence diagnostic results seem satisfactory.

### E.1 Autocorrelation function

It is a common practice to calculate first the autocorrelation function in order to measure the independence of the sequence of the draws. Figure 7 depicts the autocorrelation at lags 1, 5, 10 and 50. Autocorrelation of the VAR ($\alpha_t$) and contemporaneous coefficients ($b_t$) are smaller than 0.2 except for some exceptions, indicating independence of the sequence of the draws. The same conclusion can be made for the standard error coefficients where the autocorrelation at lags 5, 10 and 50 are smaller than 0.6 (for the majority, the plots are below 0.4).

Figure 7: Autocorrelation diagnostic



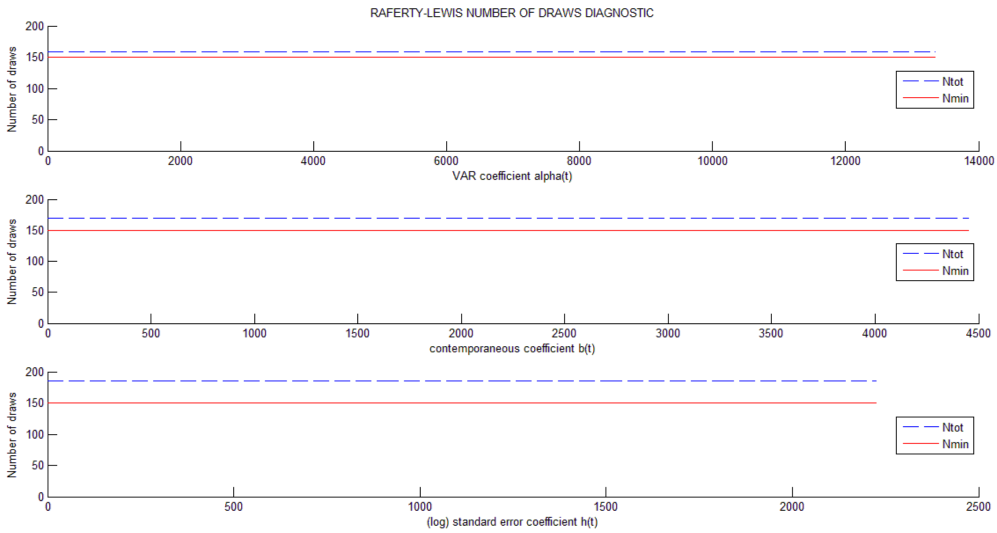

*Source: Authors calculation*

[11] We used a modified Matlab version code of James P. Lesage to implement the convergence diagnostic tests.

## E.2 Raferty-Lewis diagnostics

Raferty and Lewis (1995)[12] proposed a diagnostic test that permits to calculate the total number of draws needed to achieve a certain level of accuracy (Ntot) and the minimum number of draws that would be needed if the draws represented an i.i.d chain (Nmin). Raferty and Lewis number of draws in Figure 8 indicates that for all coefficients, the total number of draws needed to achieve a certain degree of accuracy is less than 200.

Figure 8: Raferty-Lewis "Number of draws" diagnostic

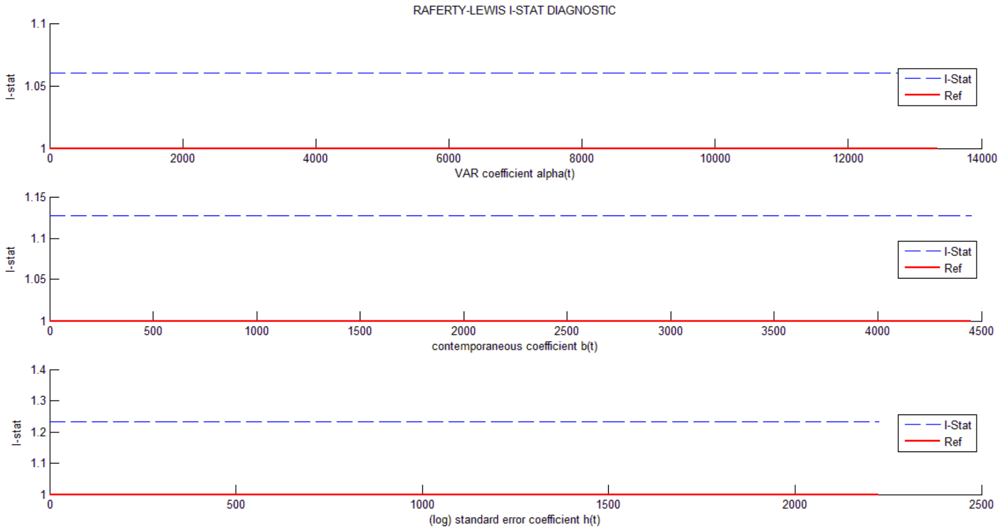



*Source: Authors calculation*

The Raferty and Lewis (1995) I-statistic is the ratio of the total number of draws needed to acheive a certain level of accuracy (Ntot) to the minimum number of draws that would be needed if the draws represented an i.i.d chain (Nmin). Values of I-stat exceeding 5 indicate a convergence problem. Figure 9 indicates that the Raferty and Lewis I-STAT for all coefficients are less than 2.

[12] Parameters value used to implement the Raferty and Lewis (1995) diagnostic tests are defined as follows : the quintile of the marginal posteriori is set to be equal to 0.025, the minimum probability needed to achieve the required accuracy to 0.95, and the desired accuracy to 0.005.

Figure 9: Raferty-Lewis "I-STAT" diagnostic




*Source: Authors calculation*