



# Growth Propensities under Capitalism and Profit-Oriented Market Socialism

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#### Abstract

This research mathematically investigates the hypothesis of slow economic growth under capitalism, relative to potential economic growth under a profit-oriented market socialist alternative. The hypothesis is examined within two different economic models: (1) a current-period model that looks at the optimal level of investment in any period from the respective standpoints of the capitalist minority and the general population, and (2) a steady-state equilibrium model that looks at the optimal capital-labor ratio from the respective standpoints of the capitalist minority and the general population. In both cases, two inequalities are derived for the determination of parameter combinations under which growth retardation will hold, the first under the assumption that the aggregate CES production function is linear homogeneous, and the second under the assumption that this function is homogeneous to any degree. The fact that parameter combinations exist under which growth retardation would hold, and also under which growth retardation would not hold, suggests that this issue is essentially an empirical issue rather than a theoretical issue.

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#### 1. Introduction

Prior to the seminal conceptual work of Oskar Lange enunciated in his essay "On the Economic Theory of Socialism" (1938), both orthodox economists and orthodox socialists would have summarily dismissed the notion of "market socialism" as a veritably ludicrous oxymoron. Lange's credentials as an accomplished neoclassical theoretician, however, lent considerable weight to his argument that a market socialist economic system, characterized by public ownership of the preponderance of capital utilized within large-scale corporate business enterprise, could successfully mimic the economic performance of a hypothetical perfectly competitive capitalist economy. In Lange's view, this gave the edge to market socialism since the real-world capitalist economy, as opposed to the hypothetical textbook capitalist economy, was (and remains) actually dominated by imperfectly competitive mega-corporations.

Although Lange's central complaint against the contemporary capitalist economy is its microeconomic inefficiency owing to the breakdown of perfect competition (a static problem), writing as he did against the background of the Great Depression, he took it for granted that capitalism was also subject to the dynamic problem of long-term sub-optimal economic growth. The apparent success of Keynesian stabilization policy in preventing disastrous business downturns since the end of World War II, however, has restored most of the

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pre-Great Depression confidence in the overall economic performance of capitalism, static and dynamic—at least among the majority of contemporary mainstream economists.

The level of enthusiasm among this majority for the economic performance of contemporary capitalism fluctuates with the condition of the economy, and has naturally been significantly reduced recently owing to worldwide economic vicissitudes throughout much of the 2000s. But as serious as they may be, these vicissitudes have apparently not seriously undermined overall mainstream support for the fundamental institutional status quo. Nevertheless, the possibility remains of a market socialist alternative to the capitalist status quo that would exhibit better performance, not only in terms of economic equality but also in terms of static efficiency and dynamic growth. With respect to dynamic performance, the suppression of serious depressions does not necessarily rule out the possibility that the economy is growing at less than the optimal rate, and the possibility exists that the optimal rate could indeed be achieved via some sort of social intervention—as, for example, through the establishment of a market socialist economy.

The possibility of a "slow growth problem" under contemporary capitalism has indeed been considered by a number of economists such as Thurow (1987), Krugman (1987), Maddison (1987), Silk (1995), Lawrence (1996), Sachs and Warner (1997), Azam et al (2002), Sato (2002), Ibarra (2003), Mauro (2004), Hein and Truger (2005), Gibson (2007), and Joffe (2011). Many of these authors tend to attribute the blame, depending on their ideological predispositions, either to misguided public policies or to imperfectly competitive big business.

Additional concern over possible slow growth has arisen out of the substantial "new growth theory" literature based on the seminal contributions of Paul M. Romer (1986, 1990): see, for example, Saint-Paul and Verdier (1997), Nelson (1998), Ruttan (1998), Lutz (1999), Roe and Mohtadi (2001), Salvadori (2003), Rima (2004), Bhaduri (2006), Falvey et al (2006), and Durlauf, Kourtellos and Tan (2008). Whereas the "old" neoclassical growth theory tends to assume that both the saving and technological growth rates are exogenous, the "new" theory postulates their endogeneity. More specifically, it has been argued that there are both increasing returns and external returns to technical knowledge, with the consequence that the market capitalist economy under-invests in scientific human capital and scientific research. Another perceived problem has to do with the high level of economic inequality under contemporary capitalism. Of course, the traditional economic hypothesis has been that economic inequality, in and of itself, fosters growth by putting more income at the disposal of wealthy households with higher marginal propensity to save (Kaldor, 1955). More recently, however, several economists have raised serious questions about this hypothesis on both statistical and theoretical grounds: see, for example, Galor and Zeira (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996), Banerjee and Duflo (2003), Milanovic (2011), Berg and Ostry (2011). But whether based on "new growth theory," the high level of economic inequality under contemporary capitalism, or other considerations, the problem of "slow growth" as perceived by these various mainstream economists, to the extent that it exists, may presumably be addressed by measures short of any fundamental socio-economic reorganization (e.g., more income redistribution, or more discretionary government support of scientific education and research).

But that small minority of economists who retain a sympathetic interest in Lange's brainchild of market socialism tend to look for deeper factors that are imbedded in the institutional nature of capitalism itself: specifically, the division of society into a minority class of capitalists primarily dependent on capital property income, and a majority class of workers who received little or no capital property income. This paper explores a mathematical approach to the question of slow growth (or "growth retardation") under capitalism as a consequence of the system's fundamental nature.

This investigation is motivated particularly by extensive work on "profitoriented market socialism," an evolution from the classic market socialism model of Oskar Lange. Profit-oriented market socialism would dispense with the technical elements of Lange's blueprint (the CPB centralized pricing system, the production rules, and the industry authority approach to investment allocation) on grounds that these elements represent an impractical attempt to reproduce in the real world the theoretically ideal Marshallian-Walrasian equilibrium. There are currently at least three separate and distinct proposed blueprints for profitoriented market socialism in the systems literature: those of Leland Stauber (1975, 1977, 1987, 1993), John Roemer and co-authors (1991, 1992, 1993, 1994), and James Yunker (1992, 1993, 1995, 1997, 2001, 2007)

Under profit-oriented market socialism, the publicly owned business corporations would be instructed to maximize long-term profitability, and their executives would be motivated to pursue this objective by the same sort of incentives currently operative under capitalism. The abandonment of marginal cost pricing would eliminate the possibility of achieving optimality in terms of microeconomic efficiency (assuming that the contemporary real-world economy approximates imperfect competition more closely than it approximates perfect competition), but this disadvantage would be counter-balanced by considerable reduction of the sort of incentive and monitoring problems perceived by numerous critics of Lange's plan, among them Hayek (1940), Bergson (1948, 1964), Roberts (1971), Feiwel (1972), Nutter (1974), Vaughn (1980), Murrell (1983), Milenkovitch (1984), Lavoie (1985), Steele (1992), Keren (1993), Makowski and Ostroy (1993), Shleifer and Vishny (1994), Milonakis (2003), Hanousek and Filer (2009), and de Soto (2010). Under profit-oriented market socialism, business enterprise would be conducted almost precisely as it is currently under capitalism: the difference would lie not in the production but rather in the distribution of capital property return (dividends, interest and so on). The principal difference between profit-oriented market socialism and contemporary capitalism would be the payment of capital property return to the general public as a social dividend (possibly in proportion to labor income, or possibly equally), rather than to individual capital owners on the basis of financial capital ownership.

Although advocates of profit-oriented market socialism such as Stauber, Roemer and Yunker perceive the primary advantage of market socialism over capitalism to lie in a more equal and equitable distribution of capital property return, these advocates also claim various static efficiency and dynamic progressiveness advantages for their proposed systems-although obviously, the attainment of Pareto optimality through marginal cost pricing cannot be one of these. In the area of investment and growth, they argue that certain improvements would become possible since the general interest of society as a whole would take precedence over the private interests of the minority capitalist class. Improvements would be possible in terms of both the size of the investment flow, and its allocation over alternative uses. Stauber, Roemer and Yunker all perceive the likelihood that under market socialism there would be a public allocation to business capital investment to supplement the flow of private household saving devoted to this purpose, and that the total amount allocated to business capital investment would be larger than it is under capitalism. Business capital investment in this context includes research and development as well as plant and machinery.

As is well known, there is no clear-cut case of market socialism anywhere in the real world. If there were, proponents could cite its favorable characteristics, while opponents could point out its faults and liabilities. However, if there were indeed one important nation in the contemporary world that is at least a distant cousin to the profit-oriented market socialism envisioned by Stauber, Roemer and Yunker, it would have to be the People's Republic of China. As everyone knows, the PRC has experienced remarkable economic growth since it abandoned the last vestiges of Stalinist central planning in the 1980s. China shares with India the characteristic of a very large population size in conjunction with a relatively limited natural resource base. A principal difference between the two nations is that China's economy is closer to market socialism while India's is closer to market capitalism. China's recent economic growth has been significantly higher than India's. For example, according to World Bank statistics on per capita income in purchasing power parity terms, China's average annual growth rate from 1980 through 2010 was 8.62 percent, while India's was 4.24 percent. Despite this, in arguing the potential advantages of market socialism over market capitalism, Stauber, Roemer and Yunker have not cited China's economic lead over India. Quite possibly this is because they are at pains to argue that market socialism is compatible with political democracy, while the deficiencies of China in this respect cannot be overlooked.

The question addressed here is whether the level of capital investment and capital stock that is optimal from the standpoint of the general population is in

fact larger than the level of capital investment and capital stock that is optimal from the standpoint of the minority capitalist class. In order to sharpen the analysis, an explicit function approach is adopted. The thrust of the analysis is toward the conclusion that this is an empirical question and not a theoretical question. That is to say, according to relatively conventional neoclassical specifications of the problem, the level of capital investment and capital stock under contemporary capitalism (i.e., the level preferred by the minority capitalist class) could be either lower than or higher than they would be under market socialism (i.e., the level that would be preferred by the general population). According to the analysis, the outcome depends on the numerical values of certain key parameters. If ambiguity is the result under an explicit function approach, obviously the ambiguity could not be less under a general function approach. Moreover, if ambiguity is the outcome using a "simple" neoclassical economic model, it seems likely that the outcome would be the same using a more "advanced" model-assuming that the advanced model incorporates all the assumptions of the simple model without altering or deleting any of them. Of course it is not impossible that a more complicated model that builds directly on a simpler model could eliminate ambiguous results from the simpler model, but this seems more likely to be the exception rather than the rule.

Certainly the most elaborate formal examination of this particular question in the economic literature to date has been a modest theoretical literature on "the dynamic inefficiency of capitalism" inspired by Kelvin Lancaster's 1973 paper of the same name: for example, King and Ferguson (1983), Pohjola (1983), Galor (1986), Rhee (1991), Noh (1992), Gibson (2003), Gutiérrez (2008), and Antonelli and Teubal (2008). The basis of the problem perceived by Lancaster is that while the general population (i.e., the "workers" since most of the general population depends mostly on labor income) determines the level of ex ante saving, it is the "capitalists" (i.e., that small fraction of the general population whose income is mostly derived from capital property) who determine, through their control of business enterprise, how much ex ante saving is converted into ex post investment. The "workers" and the "capitalists," assumed to be mutually exclusive and exhaustive categories, desire to maximize their respective consumptions over a finite time horizon. The problem sets up as a "differential game" with both sides adjusting their behavior to take into account the selfinterested behavior of the other side. Not surprisingly, the solution of the game, in general, is different from the social welfare optimum according to which the proportion of income allocated to saving and the proportion of saving allocated to investment would be determined to maximize the sum of worker consumption and capitalist consumption.

In order to make this somewhat subtle optimization scenario analytically tractable, some severely restrictive assumptions are required: among other things, that the workers unilaterally determine, within fixed upper and lower limits, what proportion of total output they receive (an assumption inconsistent with marginal product factor pricing), and that total output is proportional to capital stock (an assumption inconsistent with the fundamental economic principle of diminishing returns to a factor of production). Even with these restrictions, the analysis is cumbersome and somewhat unworldly, as, for example, in the division of the planning period into two intervals with qualitatively different characteristics (the "bang-bang" solution). Moreover, the conclusion drawn from the effort is only that actual capital accumulation will be different from socially optimal accumulation: not that it will be either lower than the optimum (retarded growth) or higher than the optimum (over-stimulated growth).

Our objective here will be a somewhat more down-to-earth analysis of the problem based upon two economic models, both of which are characterized by a mathematically explicit aggregate production function widely applied in contemporary economics: the Constant Elasticity of Substitution (or CES) form. The first model may be termed a "current-period" model, although it should be noted that since the level of investment in the current period affects the output in the current period through its effect on capital, the "current period" in this model is fairly lengthy, possibly on the order of five to ten years. As a corollary to this, it is also worthwhile to point out that the definition of "capital" in this context is not necessarily confined to directly productive plant and equipment. In the broader sense, capital refers the cumulative value of all investments, including those in research and development. The second model is based on the wellknown steady-state paradigm of economic growth theory. Under both models, explicit solutions are obtained both for the levels of investment and capital preferred by the capitalist minority, and for those preferred by the general public. With these explicit solutions in hand, we can then determine what parameter combinations are consistent with the hypothesis of growth retardation under capitalism, and what combinations are inconsistent with this hypothesis. The current-period model is examined in Section 2, and the steady-state model in Section 3. A brief summary and conclusion is provided in Section 4.

## 2. Current-Period Analysis

The following analysis of the growth retardation hypothesis is based on a number of strong assumptions, as follows: (1) the aggregate labor supply is exogenously determined and there is full employment of labor; (2) capitalists receive the value of total output less the marginal product payment to aggregate labor (in the case of diminishing returns to scale, this value would be greater than the marginal product payment to capital); (3) all production activity in the economy may be encapsulated in a single aggregate production function relating an undifferentiated aggregate output Y to undifferentiated aggregate capital K and undifferentiated aggregate labor L; (4) all capitalists in the economy may be encapsulated in a single representative capitalist, and all other individuals in the economy (the "general public") may be encapsulated in a single representative capitalist desires to maximize capitalist consumption, assumed to be a constant proportion of capitalist income; (6) the

representative individual desires to maximize aggregate consumption, defined as aggregate output less investment (this assumes the Keynesian equilibrium condition S = I); (7) the rate of capital depreciation is a constant; (8) the aggregate production function is a Constant Elasticity of Substitution (CES) form. Initially we will assume that the CES production function is linear homogeneous, later relaxing this assumption to allow for any degree of homogeneity.

The aggregate production function is defined as follows:

$$Y = F(K,L) = (\alpha K^{-\rho} + \beta L^{-\rho})^{-1/\rho} = (\alpha ((1-\delta)K_o + I)^{-\rho} + \beta L^{-\rho})^{-1/\rho}$$
(1)

where Y is output, K is final capital (defined as initial, or beginning-of-period, capital  $K_0$ , less depreciation  $\delta K_0$  of initial capital, plus investment I), L is labor, and  $\alpha$ ,  $\beta$ ,  $\rho$  and  $\delta$  are parameters. The rate of capital depreciation is  $\delta$ . The elasticity of substitution between capital and labor is  $\sigma = 1/(1+\rho)$ . If the elasticity of substitution were equal to one, the CES production function would reduce to the special-case Cobb-Douglas production function:  $Y = K^{\alpha}L^{\beta}$ , in which case  $\alpha$  and  $\beta$  would be respectively the output elasticity of capital and the output elasticity of labor. Because of this relationship, we will refer to  $\alpha$  as the "quasioutput elasticity" of capital and to  $\beta$  as the "quasi-output elasticity" of labor. Now define the amount of income going to capital K as its marginal product payment, or alternatively as the residual after labor income has been deducted from total income:

$$A^{K} = F_{K}K = Y - F_{L}L \tag{2}$$

The objective of the capitalist class is to maximize its consumption, assumed proportional to its income:  $C^{K} = (1 - s)A^{K}$  where s is the capitalist saving rate. Setting the first derivative of  $C^{K}$  with respect to I equal to zero and solving for I results in the optimal level of investment from the standpoint of the capitalist minority:

$$I^* = \left(\frac{\alpha}{\beta\rho}\right)^{1/\rho} L - (1-\delta)K_o \tag{3}$$

Note that a necessary condition for this result to be mathematically meaningful is that  $\rho$  be greater than zero, otherwise the parenthesized expression

would be negative, and a negative number raised to a non-integer power is not a real number. If  $\rho$  is greater than zero, then from the relationship between this parameter and the elasticity of substitution  $\sigma$ ,  $\sigma$  is less than one. Thus a necessary condition for this optimization problem to be meaningful is that there be relatively low elasticity of substitution between capital and labor. If the elasticity of substitution is less than one, then an increase in the capital-labor ratio will reduce the proportion of total income going to capital. This is intuitively evident, since if an increase in *I* increased both the total amount of income and capital's share of income,  $\mathcal{A}^{K}$  would increase without limit with increases in *I*.

To determine whether this level of investment is sub-optimal, we compare it to optimal investment from the standpoint of the general population. Although the capitalist minority is a component of the general population, and thus its preferences are incorporated into those of the general population, it is still the case that owing to the substantial difference in terms of primary source of income (property income for capitalists versus labor income for non-capitalists), the representative individual among capitalists will have different preferences than the representative individual among non-capitalists. The general population preference might then be thought of as some kind of weighted average of capitalist and non-capitalist preferences, with the much heavier weight on the latter owing to the much larger proportion of the total population represented by non-capitalists. While recognizing that, strictly speaking, this incorporates a simplifying assumption, it is still a reasonable approximation to reality to specify as the optimal level of investment for the general population that level of investment which maximizes total consumption C:

$$C = Y - I = F(K, L) - I = F((1 - \delta)K_o + I, L) - I$$
(4)

Setting the first derivative of C with respect to I equal to zero and solving for I results in the optimal level of investment from the standpoint of the general population:

$$I^{**} = \left(\frac{\alpha^{\rho/(1+\rho)} - \alpha}{\beta}\right)^{1/\rho} L - (1-\delta)K_o$$
(5)

A situation of growth retardation is defined by  $I^* < I^{**}$ . In this case, if the general population were to save more than the amount  $I^*$ , the growth retardation hypothesis of proponents of market socialist blueprints such as Stauber, Roemer and Yunker maintains that the surplus of saving would be re-channeled into consumption loans to private households and/or government agencies. This is possible, so it may be argued, because investment in business physical capital,

either directly via retained earnings decisions of the firms themselves or indirectly via investment decisions of financial intermediaries, is controlled by the capitalist minority through their ownership rights over all types of business enterprise, financial and non-financial. A situation of growth over-stimulation is defined by  $I^* > I^{**}$ . It seems less plausible that the capitalist minority could "force" the general population to save more (in an ex post sense) than it naturally wishes to, although possibly there is some mechanism by which this could be accomplished. However, since our principal interest here is potential growth retardation rather than potential growth over-stimulation, we will forego speculation on such mechanisms.

The above results for I may be re-written in terms of the optimal currentperiod capital-labor ratio from the standpoint of the capitalist minority ( $k^*$ ) and from the standpoint of the general population ( $k^{**}$ ) as follows:

$$k^* = \frac{K^*}{L} = \frac{(1-\delta)K_o + I^*}{L} = \left(\frac{\alpha}{\beta\rho}\right)^{1/\rho}$$
(6)

$$k^{**} = \frac{K^{**}}{L} = \frac{(1-\delta)K_o + I^{**}}{L} = \left(\frac{\alpha^{\rho/(1+\rho)} - \alpha}{\beta}\right)^{1/\rho}$$
(7)

If labor L grows at a constant exponential rate g (i.e.,  $L = L_{e^{\mathcal{B}}}$ ), then in long-run steady-state equilibrium (assuming this theoretical concept is meaningful), capital and output will also grow at this rate. Prior to the economy arriving at the steady-state equilibrium, the growth rate in K and Y would be different under the two criteria; after arriving at this point the absolute values of K and Y would be different (assuming the same initial point), even though their growth rates would be the same. Using (6) and (7), we arrive at the following basic result:

$$k^* \le k^{**} \text{ as } \alpha \le (1 - \sigma)^{1/\sigma}$$
where  $\sigma = 1/(1 + \rho).$ 
(8)

Table 1 and Figure 1 provide a numerical perspective on this result. The table shows various combinations of  $\sigma$  and  $\alpha$  (obtained from  $\alpha = (1 - \sigma)^{1/\sigma}$  for  $\sigma = 0.05$  to  $\sigma = 0.95$  by increments of 0.05) for which  $k^* = k^{**}$ . Figure 1 plots these combinations. The figure is annotated to indicate the region of parameter values to the southwest of the  $k^* = k^{**}$  curve in which  $k^* < k^{**}$  (retarded

growth) and the region of parameter values to the northeast of the  $k^* = k^{**}$  curve in which  $k^* > k^{**}$  (over-stimulated growth).

The first point to be made is that the question of whether or not capitalism retards growth below its theoretical optimum would appear to be an empirical question and not a theoretical question. The model developed here does not provide an unambiguous answer on the basis of the assumptions of the model. Depending on the numerical values of the two key parameters  $\alpha$  (quasi-output elasticity of capital) and  $\sigma$  (elasticity of substitution between capital and labor), it could go either way. Given the general thrust of contemporary theoretical economics (i.e., that "anything is possible"), this indication might not seem all that implausible. However, the issue of socialism versus capitalism is perhaps the most ideologically loaded policy issue with which economics has ever been concerned. There is a strong tendency for those on both sides of the debate to harbor strong a priori presumptions concerning the relative performance of the two systems, and thus to be skeptical of any theoretical analyses that do not unambiguously support these presumptions. This is an issue on which it is rather difficult for the average economist to maintain the ideal level of impartiality and objectivity.

The second point to be made is that on the basis of the commonly accepted ballpark estimates of  $\alpha$  and  $\sigma$ , the likelihood of real-world growth retardation under capitalism would seem to be small. Interest in statistical estimation of aggregate production functions was at its peak almost a half-century ago, and from that era two of the most enduring empirical regularities were that the elasticity of substitution between capital and labor is approximately unity or perhaps a bit less, and that the output elasticity of capital (in the Cobb-Douglas special case of the CES production function) is in the order of 0.25. As can be seen from Table 1, if  $\alpha$  were 0.25, the elasticity of substitution would have to be less than 0.50 for growth retardation to take place. This low an elasticity of substitution is certainly not consistent with the ballpark estimates of 50 years ago. On the other hand, the output elasticity of capital estimated from a Cobb-Douglas specification is not necessarily an adequate estimate of the "quasi-output elasticity of capital" of the CES production function. Moreover, it need not be belabored that there are serious conceptual, data, identification and estimation problems confronting efforts to statistically estimate aggregate production functions, and that consequently such "empirical regularities" as we currently entertain are certainly not overly reliable.

We continue the analysis now by relaxing the assumption that the production function is linear homogeneous. Letting  $\nu$  indicate the degree of homogeneity, the production function becomes:

$$Y = F(K,L) = (\alpha K^{-\rho} + \beta L^{-\rho})^{-\nu/\rho} = (\alpha ((1-\delta)K_{\rho} + I)^{-\rho} + \beta L^{-\rho})^{-\nu/\rho}$$
(9)

If  $v \neq 1$ , then the "factor exhaustion theorem" stating that the sum of marginal product factor payments equals output  $(F_KK + F_LL = Y)$  does not hold. If v < 1 then the sum of marginal product factor payments is less than output  $(F_KK + F_LL < Y)$ , while if v > 1 the sum of marginal product factor payments is greater than output  $(F_KK + F_LL > Y)$ . Thus if v < 1, the capitalists collect not only their own marginal product payment, but also the positive residual between output and the sum of marginal product factor payments. On the other hand, if v > 1, this residual is negative and the capitalists collect only output value less the marginal product payment to labor, which is less than their own potential marginal product payment. In other words, the operative criterion variable is not  $C^K = (1-s)F_KK$  but rather  $C^K = (1-s)(Y - F_LL)$ .

Setting the first derivative of  $C^{K}$  with respect to I equal to zero and solving for I results in the optimal level of investment from the standpoint of the capitalist minority:

$$I^* = \left(\frac{\alpha}{\beta(\rho+\nu-1)}\right)^{1/\rho} L - (1-\delta)K_o$$
(10)

Note that if  $\nu = 1$ , this reduces to the same  $I^*$  shown in equation (3) above. Note also that a necessary condition for existence is that  $\rho + \nu - 1 > 0$ .

The first-order condition for the maximization of C = Y - I with respect to I is not explicitly solvable for  $I^{**}$ . However, we can determine whether  $I^{**} > I^*$  by substituting into the derivative of C with respect to I the expression for  $I^*$  given in equation (10) above. If the derivative evaluated at this point is greater than zero, this shows that at that investment level, the economy is still on the upward-sloping part of the dome-shaped function relating I to C. Therefore, by manipulating the inequality  $dC/dI \ge 0$  at  $I = I^*$ , we arrive at the following result:

$$I^* \le I^{**} \text{ as } \alpha \le \beta^{1-\nu} \nu^{\rho} (\rho + \nu)^{-\rho - \nu} (\rho + \nu - 1)^{1+\rho} L^{\rho(\nu - 1)}$$
(11)

By evaluating the right-hand side of this expression for  $\nu = 1$ , and substituting  $\rho = (1/\sigma) - 1$ , we find that result (11) is consistent with result (8).

Without loss of generality, we may set L = 1 and thus disregard the last term in the RHS of the inequality expression for  $\alpha$ . If  $\nu \neq 1$ , then aside from  $\nu$ itself, the value of  $\beta$  enters into the empirical question of growth retardation. A higher  $\beta$  value would increase the right-hand side of the inequality and hence the likelihood of growth retardation. However, the effect of  $\beta$  within a reasonable range of variation is numerically small compared to that of  $\nu$  for a similarly reasonable range of variation. Table 2 is analogous to Table 1 but evaluates the  $\alpha$ - $\sigma$  function for five values of  $\nu$  1.2, 1.1, 1.0, 0.9 and 0.8, with  $\beta$  set at 0.7. For sufficiently high values of  $\sigma$  and  $\nu$ ,  $\alpha$  is undefined because of violation of the  $\rho + \nu - 1 > 0$  condition: these cells in the table are denoted NC for "not computable." Figure 2 plots these functions for  $\nu = 1.2$ , 1.0 and 0.8. Table 3 evaluates the  $\alpha$ - $\sigma$  function for five values of  $\beta$ : 0.5, 0.6, 0.7, 0.8 and 0.9, with  $\nu$ set at 0.9. In this case the  $\sigma$  values only go to 0.90 because at  $\sigma = 0.95$  there is a violation of the  $\rho + \nu - 1 > 0$  condition. Figure 3 is the corresponding plot. Since the plots are so close together in Figure 3, no attempt is made to differentiate them by labels indicating the corresponding  $\beta$  value. As is apparent from this tabular and graphical information, it is the case that, *ceteris paribus*, higher values of either  $\nu$  or  $\beta$  make growth retardation more likely. But the numerical effect is far more substantial in the case of the  $\nu$  parameter.

The numerical degree of homogeneity indicates the type of returns to scale: thus if v > 1 there are increasing returns to scale, if v = 1 there are constant returns to scale, and if v < 1 there are decreasing returns to scale. As is well known, increasing returns to scale in a particular industry rules out a perfectly competitive equilibrium in that industry. Lange believed that most industries in the real world are actually imperfectly competitive, and the prevalence of increasing returns to scale is one obvious way to account for this situation. It might seem intuitively plausible that if most industries in an economy have production functions characterized by increasing returns to scale, then the aggregate production function would also have increasing returns to scale. But it has to be acknowledged that such a conclusion assumes away very complex aggregation issues. In fact, it is by no means certain that any sort of aggregate production function for the entire economy is analytically meaningful, let alone one characterized by increasing returns to scale. With this in mind, the following should be considered as a technical observation only, whose real-world significance is uncertain: From the results in Table 2 and Figure 2, it is apparent that if there are increasing returns to scale, then even if  $\sigma$  is approximately unity (consistent with its "ballpark" value from the high era of aggregate production function estimation), quite a substantial value of  $\alpha$  could still be consistent with growth retardation. From the results in Table 2 and Figure 2, it is apparent that if there are increasing returns to scale, then even if  $\sigma$  is approximately unity (consistent with its "ballpark" value from the high era of aggregate production function estimation), quite a substantial value of  $\alpha$  could still be consistent with growth retardation. For example, if v were 1.2 and  $\sigma$  were 0.90, then for all values of  $\alpha$  up to 0.20995, growth retardation would hold.

Be that as it may, it is doubtful whether the possibility of increasing returns to scale adds all that much to the likelihood that growth retardation under capitalism is an existent, real-world phenomenon. Going back once more to the high era of aggregate production function estimation, few estimations from that period suggest that the degree of homogeneity of the aggregate production function—presuming that the aggregate production is both meaningful and homogeneous—is significantly in excess of unity. There is also the fact that on the basis of intuition it seems likely that if economies of scale were extremely important in the real world, then we would witness, for most industries, a much smaller number of very large firms.

## 3. Steady-State Analysis

According to the well-known Solow-Swan model of steady-state economic growth (Solow, 1956; Swan, 1956), assuming a linear homogeneous neoclassical production function in capital and labor, and the absence of technological progress, the steady-state capital-labor ratio is determined by:

$$sf(k) = (\delta + g)k \tag{12}$$

where *s* is the rate of saving, f(k) is the per capita form of the production function (per worker output *y* as a function of per worker capital *k*),  $\delta$  is the rate of capital depreciation, and *g* is the rate of growth of the labor force. From this result, we see that at long-run steady-state equilibrium the saving rate is  $s = (\delta + g)k/f(k)$ .

According to the conventional "Golden Rule" principle in optimal growth theory (Phelps, 1966), k should be set at the level that maximizes long-run steady-state per worker consumption, as defined by c = f(k) - sf(k). This criterion is analogous to the criterion specified for the "general public" in the preceding section: C = Y - S = Y - I. If the steady-state saving rate is substituted into the expression for steady-state per capita consumption, we have  $c = f(k) - (\delta - g)k$ . The first-order condition for the maximization of c with respect to k is then:

$$f'(k) = \delta + g \tag{13}$$

Using notation analogous to that of the previous section, we will denote as  $k^{**}$  the k that satisfies this condition. The above specification of the Golden Rule first-order condition follows the original presentation of Phelps (1966, p. 10), in which the first derivative of c = f(k) - sf(k) with respect to k is equated to zero. A common alternative is to utilize the relationship between s and k shown by (12) above to obtain k as a function of s, substitute this into the equation for c, and maximize with respect to s, thus obtaining the Golden Rule saving rate. The two methods are equivalent, and we utilize maximization of c with respect to k

here to better clarify the analogy between the current-period results and the steady-state results.

In contrast to the Golden Rule solution, a reasonable long-run steady-state objective for the capitalist minority is the maximization of net profits after depreciation:  $\pi = f'(k)k - \delta k$ . Under the assumption of linear homogeneity in the production function, this is analogous to the capitalist criterion proposed in the current-period model of the previous section:  $A^K = F_K K$ . Since the capitalist saving rate *s* is constant, maximization of  $C^K$  is equivalent to maximization of  $A^K$ . The two models handle depreciation differently, and in the case that the degree of homogeneity is not unity, in the current-period model the alternative  $A^K = Y - F_L L$  criterion supersedes the  $A^K = F_K K$  criterion. The first-order condition for the maximization of  $\pi$  with respect to *k* in the steady-state model is as follows:

$$f'(k) = \delta - f''(k)k \tag{14}$$

Using notation analogous to that of the previous section, we will denote as  $k^*$  the k that satisfies this condition: this particular k will produce for capitalists a "Golden Age" during which their net profit income will be maximized.

Using the standard neoclassical assumptions (f'(k)>0 and f''(k)<0), we can immediately discern one set of conditions that are sufficient for growth retardation to hold. This is the case where depreciation is relevant ( $\delta > 0$ ), but there is zero population growth (g = 0). If the RHS of (13) consists of  $\delta$  alone, then it is unambiguous that the RHS of (14) is greater than the RHS of (13), and hence that  $k^* < k^{**}$ . On the other hand, under positive population growth with  $\delta > 0$ ,  $k^* < k^{**}$  only if  $\delta - f''(k)k > \delta + g$ . It is certainly possible that this latter inequality will hold true and that consequently growth retardation will hold. But there is no compelling *a priori* theoretical argument that can be made to this effect. The situation is basically ambiguous, and the same is true if we ignore the positive depreciation possibility by setting  $\delta = 0$ . If  $\delta = 0$ , then  $k^* < k^{**}$  only if -f''(k)k > g.

Additional illumination on the growth retardation hypothesis within the context of steady-state equilibrium may be obtained by applying an explicit mathematical form. Assuming linear homogeneity in the production function (constant returns to scale), the per capita version of the CES aggregate production function used in the previous section is as follows:

$$y = f(k) = (\alpha k^{-\rho} + \beta)^{-1/\rho}$$
(15)

Applying this mathematical form in (13), we may solve explicitly for the capital-labor ratio k that maximizes steady-state per capita consumption c (the Golden Rule k):

$$k^{**} = \left(\frac{\left(\frac{\alpha}{\delta+g}\right)^{\rho/(1+\rho)} - \alpha}{\beta}\right)^{1/\rho}$$
(16)

Comparing this result to (7) above, which gives  $k^{**}$  for the current-period model, we see that the results are analogous and that the single exception is that the  $\alpha^{\rho/(1+\rho)}$  term in the numerator of (7) has been replaced by  $(\alpha/(\delta+g))^{\rho/(1+\rho)}$ . Since the sum of  $\delta$  and g (respectively the rate of capital depreciation and the rate of labor force growth) is likely to be considerably less than one, this suggests that the long-term steady-state  $k^{**}$  is likely to be considerably in excess of the current-period  $k^{**}$ .

If we apply the CES mathematical form (15) in result (14) which determines  $k^*$  (the Golden Age k), we are unable to solve explicitly for k unless  $\delta = 0$  (i.e., we ignore depreciation). However, in a numerically explicit case, it is easy enough to solve for  $k^*$  iteratively using a computer program. In the case that  $\delta = 0$ , we have the explicit result:

$$k^* = \left(\frac{\alpha}{\beta\rho}\right)^{1/\rho} \tag{17}$$

This result is identical to result (6) above for  $k^*$  in the current-period model. Ignoring depreciation, therefore, we see that the capital-labor ratio that maximizes capitalist income is the same as between the current-period model and the long-term steady-state model. Since (as we saw above) the steady-state  $k^{**}$  is larger than the current-period  $k^{**}$ , while at the same time the steady-state  $k^*$  is the same as the current-period  $k^*$ , this suggests that it is more likely that  $k^*$  will be less than  $k^{**}$  (growth retardation will hold) in a long-term steady-state situation than in a current-period situation. But again, there is no reason here to suppose that  $k^*$  will *inevitably* be less than  $k^{**}$  in a long-term steady-state situation.

In the previous section using the current-period model, results (6) and (7), explicit solutions for  $k^*$  and  $k^{**}$  respectively, were combined to produce result (8), an inequality that indicates those combinations of  $\alpha$  and  $\sigma$  that will produce

growth retardation and those that will not. Results (16) and (17) can be combined similarly to produce the following result:

$$k^* \le k^{**} \text{ as } \alpha \le (1 - \sigma)^{1/\sigma} g^{(\sigma - 1)/\sigma}$$
(18)

This result is analogous to result (8), with the difference being that the RHS of the inequality contains the added term  $g^{(\sigma-1)/\sigma}$ . Given that  $\sigma < 1$  and g < 1, the term  $g^{(\sigma-1)/\sigma} > 1$ . In this case the RHS of the result (18) inequality is larger than the RHS of the result (8) inequality, meaning that there is a wider range of  $\alpha$ values for which growth retardation will hold under the long-term steady-state model specification than under the current-period model specification. For example, Table 1 for the current-period model shows that if  $\sigma = 0.80$ , then for all  $\alpha$  up to 0.133748, growth retardation would hold. For  $\sigma = 0.80$ , the term  $(\sigma - 1)/\sigma$  takes on the value of -0.25. For a labor force growth rate of one percent (g = 0.01), the term  $g^{(\sigma-1)/\sigma} = 3.16227$ . The product of 0.133748 and 3.16277 is 0.422014. So growth retardation will hold in the long-term steady-state model for  $\alpha$  values up to 0.422014, a number considerably in excess of 0.133748. As mentioned in the previous section, "ballpark estimates" of  $\alpha$  from the high period of aggregate production function estimation put the figure around 0.25. If  $\alpha = 0.25$ , then given the other parameter values specified, growth retardation would hold in a long-term steady-state equilibrium.

When depreciation is taken into account, the apparent likelihood that growth retardation will hold in long-run steady-state equilibrium is still further magnified. As mentioned above, we cannot solve explicitly in this model for the Golden Age  $k^*$ . However, we can obtain an inequality result analogous to (18) by using the same technique utilized in the previous section to obtain result (11). We substitute into the derivative of  $\pi$  with respect to k (the first-order condition for the Golden Age  $k^*$ ) the Golden Rule  $k^{**}$  given by (16). If the derivative evaluated at this point is negative, this shows that at the Golden Rule  $k^{**}$ , the economy is on the downward-sloping part of the dome-shaped function relating  $\pi$  to k, from which it follows that  $k^* < k^{**}$ . By manipulating the inequality  $d\pi/dk \leq 0$  at  $k = k^{**}$ , we arrive at the following result:

$$k^* \le k^{**} \text{ as } \alpha \le \left( (1-\sigma)(g+\delta)^{\sigma-1} + \delta\sigma(g+\delta)^{\sigma-2} \right)^{1/\sigma}$$
(19)

Note that if  $\delta = 0$ , this reduces to result (18). It is apparent that the RHS of the inequality in (19) is greater than the RHS of the inequality in (18). Thus if

 $\delta > 0$  there will be a still larger range of  $\alpha$  values over which growth retardation will hold. The larger the  $\delta$  value, the larger the range of  $\alpha$  over which growth retardation holds.

For apparently plausible values of the elasticity of substitution  $\sigma$  and the labor force growth rate g, the critical  $\alpha$  is very sensitive to the value of the depreciation parameter  $\delta$ . By the "critical  $\alpha$ " we mean the  $\alpha$  that separates the range of  $\alpha$  over which growth retardation holds from the range of  $\alpha$  over which growth retardation does not hold. Table 4, illustrated by Figure 4, contains the critical  $\alpha$  values for a range of 21  $\delta$  values from 0 to 0.01, for five different values of g. 0.005, 0.010, 0.015, 0.020 and 0.025. The  $\sigma$  value utilized for these computations is 0.8. The  $\delta$  values are quite small, ranging up to a maximum of 1 percent. But even with these small  $\delta$  values, the critical  $\alpha$  values tend to be quite high, bearing in mind that owing to the principle of diminishing returns to a factor of production, the theoretical maximum value of  $\alpha$  is one. The higher the rate of labor force growth g, the smaller the critical  $\alpha$  value for any given value of  $\delta$ . But even with a very high rate of labor force growth such as g = 0.025 (2.5) percent), and a very low coefficient of depreciation such as  $\delta = 0.005$  (0.5 percent), the critical  $\alpha$  is 0.60858. That is to say, for  $\sigma = 0.8$ , g = 0.025, and  $\delta = 0.005$ , all  $\alpha$  values up to 0.60858 will cause growth retardation.

Nevertheless, the point remains that whether or not growth retardation would actually hold under long-term steady-state equilibrium is an empirical question and not a theoretical question. To show that there is a wider range of parameter values over which a certain phenomenon will occur, is not to prove that the phenomenon occurs. The parameter values still have to fall within specific ranges in order for the phenomenon to occur.

To sum up, it would appear that the likelihood of growth retardation under capitalism, while still not 100 percent, would be considerably greater within the context of long-run steady-state equilibrium than it would be under a currentperiod approach such as undertaken in the previous section of this paper. This leads to the question of which approach is more realistic-or, to those who would be inclined to scoff at the notion of growth retardation under capitalism, of which approach is less unrealistic. It need not be emphasized that most economic models are based on a variety of strong assumptions-assumptions that critics describe as dubious, restrictive, implausible, and so on. Of course, without making at least some strong assumptions, very few potentially testable indications could ever be derived. This is true of both the current-period and steady-state models examined in this research. While some economists might have a preference for the current-period approach over the steady-state approach, or vice versa, the conclusions of this research show that they are closely related, and therefore it was deemed by the author sensible and efficacious to include both.

#### 4. Summary and Conclusion

In his seminal essay on market socialism, written during the Great Depression of the 1930s, Oskar Lange (1938) expressed strong suspicion that in the long term, capitalism would be subject to slow growth (growth retardation). While the substantially improved dynamic performance of capitalism in the post-Keynesian era has restored the faith of most mainstream economists in the dynamic performance of capitalism, Lange's suspicions on this issue have been echoed by latter-day exponents of "profit-oriented market socialism" who speculate that the rate of investment in business physical capital would be higher under market socialism than under capitalism because the economic interests of the general population would take precedence over the economic interests of the capitalist minority.

This paper has investigated this question in a relatively straightforward way using the tools of neoclassical economic theory. In Section 2, a current-period approach is taken which looks at the optimal level of investment in any period from the standpoint of the capitalist minority, and from the standpoint of the general population. Section 3 goes on to examine the same question within the context of the Solow-Swan steady-state growth model. In both cases, the objective is to ascertain parametric conditions under which the growth retardation hypothesis would hold, and those under which it would not hold. The research indicates that growth retardation is not necessarily implied by either of these models. Even assuming the validity of the models (an assumption which certainly might be questioned), whether or not growth retardation under capitalism is a real-world phenomenon depends on the numerical values of key parameters, parameters for which we do not have compelling statistical estimates. The basic conclusion is therefore that the growth retardation issue-like numerous other economic policy issues-is an empirical issue and not a theoretical issue. That is to say, plausible theoretical specifications of the problem do not yield unambiguous conclusions.

The fact that parameter combinations exists under which growth retardation would hold, and also under which growth retardation would not hold, whether we use a current-period model or a long-term steady-state model, suggests that this issue, as is the case with many other issues concerning the relative performance of contemporary capitalism versus market socialism, is essentially an empirical issue rather than a theoretical issue. A finding of theoretical indeterminacy is not necessarily an admission of defeat, because in principle, sufficiently reliable empirical estimates of the relevant parameters could provide strongly probative evidence. In the absence of such empirical estimates, the alternative is further development of relevant models.

Even if the theoretical picture were much sharper than it actually is, there are some wider issues involved. One of these is the control issue. Even if it could be shown theoretically with very high certainty that the capitalist class as a whole, owing to its common interest in profit income, would be benefited by a lower level of capital investment than that preferred by the general population, there remains the question of whether the capitalists could and would recognize and implement their interests in this matter. If the capitalists could elect a single "omni-director" with absolute power over the entire economy, then obviously they might have this capability. But the dispersion of economic authority over a host of autonomous capitalists, together with competitive pressures among autonomous business enterprises, may eliminate the possibility of growth retardation becoming operative in the real world.

Lest too much weight be attached to this particular criticism, it is worthwhile to note that Karl Marx himself alleged that the problem of business depressions under capitalism could never be overcome via social intervention, using exactly this argument. But Marx's prediction of ever-worsening depressions was contradicted by events. Moreover, it is a dubious supposition that pursuit of a common interest requires tight formal organization. As a matter of fact, the applied business policy literature is filled with admonitions against "overexpansion" and "exhaustion of the market." This suggests that the same restraints on investment that are sensible from the standpoint of the entire capitalist class might also seem sensible to individual capitalists.

Yet another question, however, is whether or not anything effective could be done about the problem of growth retardation—whether within the context of a profit-oriented market socialist economy or possibly a capitalist economy with greater social intervention—even if this problem were indeed operative in the real world. After all, profit-oriented market socialism aims to mimic the existing market capitalist economy. If the mimicry is successful, the negative characteristics of capitalism will tend to be duplicated as well as the positive. Even more fundamentally, if growth retardation is in fact a "disease," it might be the case that all possible cures would be worse than the disease. For example, even leaving aside economic performance issues, the inauguration of market socialism might adversely affect the institution of political democracy.

While many conservative economic authorities are concerned over the possibility of slow economic growth, their suggestions for dealing with the problem are mainly oriented toward augmenting the supply of private saving, for example by reducing taxation of capital income, or by replacing income taxation with consumption taxation. One major flaw with such proposals lies in the fact that ex post investment is not necessarily equal to ex ante saving. Saving provides a pool of potential investment resources, but not all of this pool will necessarily be utilized by profit-maximizing business enterprises.

Advocates of profit-oriented market socialism argue that the national government should make a direct allocation out of government revenue to business capital investment, thus addressing the perceived problem directly rather than indirectly. They also argue that most likely the flow of business capital investment would be significantly larger under market socialism than it is presently under capitalism. But it is difficult to assess such arguments in the absence of any real-world market socialist economies. Certainly the real-world experience of the Soviet-bloc economies does not inspire much confidence: even in the post-Stalin era, these economies maintained extremely high rates of saving and investment—but it is generally agreed that the efficiency with which these resources were utilized was abysmal. Allocating large resources to capital investment is no guarantee that the resources will be utilized efficiently and effectively.

Short of an actual experimental implementation of profit-oriented market socialism to determine its performance empirically—with the intention of returning to capitalism should the performance level prove unsatisfactory—it seems unlikely that economic science will ever provide truly compelling evidence, one way or the other, on the growth retardation issue. In conclusion, it is perhaps worthwhile to recall that in the larger scheme of things, the "growth retardation" hypothesis is a relatively minor issue in trying to evaluate the relative economic performance of capitalism versus market socialism. The extreme inequality of capital property ownership under contemporary capitalism has been established beyond a reasonable doubt, suggesting that social dividend distribution of capital property return would be more equitable than the current distribution. On the other hand, even under profit-oriented market socialism, there are potentially very serious incentive and efficiency problems—perhaps not as serious as those that would be encountered under the Langian plan of market socialism, but still quite serious.

Be that as it may, the possibility of growth retardation under capitalism is certainly of sufficient interest in its own right to be worthy of some consideration. It is the author's hope that this analysis will provide impetus to further work on this intriguing question.

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# Appendix A - Tables

Table 1 - Combinations of $\sigma$ and $\alpha$ for which $k^* = k^{**}$
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σ	$\alpha = (1 - \sigma)^{1/\sigma}$	$\sigma$	$\alpha = (1 - \sigma)^{1/\sigma}$
0.05	0.358486	0.55	0.234141
0.10	0.348678	0.60	0.217153
0.15	0.338423	0.65	0.198869
0.20	0.327680	0.70	0.179073
0.25	0.316406	0.75	0.157490
0.30	0.304551	0.80	0.133748
0.35	0.292056	0.85	0.107324
0.40	0.278855	0.90	0.077426
0.45	0.264867	0.95	0.042707
0.50	0.250000		

Table 2 - Combinations of  $\sigma$  and  $\alpha$  for which  $I^* = I^{**}$  for Various Values of v

$\sigma$	$\alpha = \beta^{1-\nu} v^{\rho} (\rho + \nu)^{-\rho-\nu} (\rho + \nu - 1)^{1+\rho} L^{\rho(\nu-1)} \text{ for } L = 1, \beta = 0.7, \text{ and } \nu = \dots$				
	1.2	1.1	1.0	0.9	0.8
0.05	6.81320	1.69191	0.35849	0.06269	0.00865
0.10	1.24099	0.68358	0.34868	0.16211	0.06724
0.15	0.71883	0.50584	0.33842	0.21312	0.12463
0.20	0.55047	0.43288	0.32768	0.23705	0.16231
0.25	0.46933	0.39147	0.31641	0.24642	0.18355
0.30	0.42117	0.36318	0.30455	0.24720	0.19302
0.35	0.38850	0.34130	0.29206	0.24242	0.19411
0.40	0.36408	0.32281	0.27885	0.23372	0.18903
0.45	0.34438	0.30615	0.26487	0.22200	0.17920
0.50	0.32749	0.29041	0.25000	0.20778	0.16550
0.55	0.31227	0.27499	0.23414	0.19133	0.14849
0.60	0.29799	0.25949	0.21715	0.17275	0.12847
0.65	0.28414	0.24358	0.19887	0.15201	0.10557
0.70	0.27034	0.22697	0.17907	0.12896	0.07979
0.75	0.25629	0.20939	0.15749	0.10333	0.05109
0.80	0.24172	0.19053	0.13375	0.07471	0.01978
0.85	0.22636	0.17006	0.10732	0.04251	NC
0.90	0.20995	0.14750	0.07743	0.00636	NC
0.95	0.19215	0.12225	0.04271	NC	NC

Note: NC = "Not Computable"

σ	$\alpha = \beta^{1-\nu} \nu^{\rho} (\rho + \nu)^{-\rho-\nu} (\rho + \nu - 1)^{1+\rho} L^{\rho(\nu-1)} \text{ for } L = 1, \nu = 0.9, \text{ and } \beta = \dots$				
	0.5	0.6	0.7	0.8	0.9
0.05	0.06061	0.06173	0.06269	0.06353	0.06428
0.10	0.15674	0.15963	0.16211	0.16429	0.16623
0.15	0.20607	0.20986	0.21312	0.21598	0.21854
0.20	0.22921	0.23343	0.23705	0.24024	0.24308
0.25	0.23827	0.24265	0.24642	0.24973	0.25269
0.30	0.23902	0.24342	0.24720	0.25052	0.25349
0.35	0.23440	0.23871	0.24242	0.24568	0.24859
0.40	0.22598	0.23014	0.23372	0.23686	0.23966
0.45	0.21465	0.21860	0.22200	0.22498	0.22765
0.50	0.20091	0.20460	0.20778	0.21057	0.21307
0.55	0.18500	0.18841	0.19133	0.19390	0.19620
0.60	0.16704	0.17011	0.17275	0.17507	0.17715
0.65	0.14698	0.14968	0.15201	0.15405	0.15588
0.70	0.12469	0.12698	0.12896	0.13069	0.13224
0.75	0.09991	0.10175	0.10333	0.10472	0.10596
0.80	0.07224	0.07357	0.07471	0.07571	0.07661
0.85	0.04110	0.04186	0.04251	0.04308	0.04359
0.90	0.00615	0.00626	0.00636	0.00644	0.00652

Table 3 - Combinations of  $\sigma$  and  $\alpha$  for which  $I^* = I^{**}$  for Various Values of  $\beta$ 

Table 4 - Critical  $\alpha$  Values at which  $k^* = k^{**}$  for Various Combinations of  $\delta$  and g

δ	$\alpha = \left( (1 - \sigma)(g + \delta)^{\sigma - 1} + \delta \sigma (g + \delta)^{\sigma - 2} \right)^{1/\sigma} \text{ where } \sigma = 0.8$				
	g = 0.005	g = 0.010	g = 0.015	g = 0.020	g = 0.025
0.0000	0.50297	0.42295	0.38218	0.35566	0.33636
0.0005	0.72372	0.51957	0.44115	0.39709	0.36783
0.0010	0.91004	0.60857	0.49704	0.43690	0.39832
0.0015	1.06673	0.69030	0.54991	0.47511	0.42783
0.0020	1.19882	0.76525	0.59985	0.51175	0.45638
0.0025	1.31067	0.83397	0.64699	0.54685	0.48398
0.0030	1.40585	0.89701	0.69148	0.58047	0.51065
0.0035	1.48726	0.95489	0.73347	0.61267	0.53642
0.0040	1.55722	1.00808	0.77311	0.64350	0.56131
0.0045	1.61759	1.05704	0.81054	0.67301	0.58536
0.0050	1.66989	1.10214	0.84590	0.70128	0.60858
0.0055	1.71537	1.14374	0.87931	0.72836	0.63101
0.0060	1.75503	1.18218	0.91091	0.75429	0.65268
0.0065	1.78970	1.21772	0.94080	0.77914	0.67361
0.0070	1.82008	1.25063	0.96909	0.80296	0.69383
0.0075	1.84675	1.28114	0.99589	0.82580	0.71337
0.0080	1.87020	1.30946	1.02129	0.84769	0.73225
0.0085	1.89084	1.33576	1.04537	0.86870	0.75050
0.0090	1.90901	1.36022	1.06822	0.88886	0.76814
0.0095	1.92502	1.38299	1.08990	0.90821	0.78520
0.0100	1.93912	1.40421	1.11050	0.92678	0.80170

## Appendix B - Figures



Figure 1 - Combinations of  $\sigma$  and  $\alpha$  for which  $k^* = k^{**}$ 

Figure 2 - Combinations of  $\sigma$  and  $\alpha$  for which  $I^* = I^{**}$  for Various Values of v







Figure 4 - Critical  $\alpha$  Values at which  $k^* = k^{**}$  for Various Combinations of  $\delta$  and g

